Accelerated Pre-Calculus/Pre-Calculus

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Packet #2

Topics Covered:

Test 8 – Exponential and Logarithmic Functions

Test 9 – Introduction to Calculus

This packet consists of daily Notes with examples and practice problems. Solutions to these worksheets are found on "Schoology." There are 2 Tests in this packet. You are to take these 2 tests and submit for grading. Your scores on these 2 tests will determine the number of awarded points (out of 5 max points).

Example 1: At their closest points, Mars and Earth are approximately 7.5×10^7 kilometers apart. Write this distance in standard form.

Properties of Exponents

Suppose m and n are positive integers and a and b are real numbers. Then the following properties hold.

Property	Definition	Example
Product	$a^m a^n = a^{m+n}$	$16^3 \cdot 16^7 = 16^{3+7}$ or 16^{10}
Power of a Power	$(a^m)^n = a^{mn}$	$(9^3)^2 = 9^{3\cdot 2}$ or 9^6
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$	$\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$ or $\frac{243}{1024}$
Power of a Product	$(ab)^m = a^m b^m$	$(5x)^3 = 5^3 \cdot x^3$ or $125x^3$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	$\frac{15^6}{15^2} = 15^{6-2}$ or 15^4

Example 2: Evaluate each expression.

(a) $\dfrac{2^4 \cdot 2^8}{2^5}$

(b) $\left(\dfrac{2}{5}\right)^{-1}$

Example 3: Simplify each expression.

(a) $(s^2t^3)^5$

(b) $\dfrac{x^3y}{(x^4)^3}$

Definition of $b^{\frac{1}{n}}$

For any real number $b \geq 0$ and any integer $n > 1$,

$$b^{\frac{1}{n}} = \sqrt[n]{b}.$$

This also holds when $b < 0$ and n is odd.

Example 4: Evaluate each expression.

(a) $125^{\frac{1}{3}}$

(b) $\sqrt{14}\cdot\sqrt{7}$

Example 5: Simplify each expression.

(a) $(81c^4)^{\frac{1}{4}}$

(b) $\sqrt[6]{9x^3}$

Rational Exponents

For any nonzero number b, and any integers m and n with $n > 1$, and m and n have no common factors

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

except where $\sqrt[n]{b}$ is not a real number.

Example 6: Evaluate each expression.

(a) $625^{\frac{3}{4}}$

(b) $\dfrac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}}$

Example 6:

(a) Express each using rational exponents $\sqrt[3]{64s^9t^{15}}$

(b) Express $12x^{\frac{2}{3}}y^{\frac{1}{2}}$ using a radical

Example 8: Simplify $\sqrt{r^7s^{25}t^3}$

Example 9: Solve $734 = x^{\frac{3}{4}} + 5$

Class work

Evaluate each expression.

20. $(-6)^{-4}$ **21.** -6^{-4} **22.** $(5 \cdot 3)^2$ **23.** $\frac{2^4}{2^{-1}}$

24. $\left(\frac{7}{8}\right)^{-3}$ **25.** $(3^{-1} + 3^{-2})^{-1}$ **26.** $81^{\frac{1}{2}}$ **27.** $729^{\frac{1}{3}}$

28. $\frac{27}{27^{\frac{2}{3}}}$ **29.** $2^{\frac{1}{2}} \cdot 12^{\frac{1}{2}}$ **30.** $64^{\frac{1}{12}}$ **31.** $16^{-\frac{1}{4}}$

32. $\frac{(3^7)(9^4)}{\sqrt{27^6}}$ **33.** $\left(\sqrt[3]{216}\right)^2$ **34.** $81^{\frac{1}{2}} - 81^{-\frac{1}{2}}$ **35.** $\frac{1}{\sqrt[7]{(-128)^4}}$

Simplify each expression.

36. $(3n^2)^3$ 37. $(y^2)^{-4} \cdot y^8$ 38. $(4y^4)^{\frac{3}{2}}$ 39. $(27p^3q^6r^{-1})^{\frac{1}{3}}$

40. $[(2x)^4]^{-2}$ 41. $(36x^6)^{\frac{1}{2}}$ 42. $\left(\frac{b^{2n}}{b^{-2n}}\right)^{\frac{1}{2}}$ 43. $\frac{2n}{4n^{\frac{1}{3}}}$

44. $(3m^{\frac{1}{2}} \cdot 27n^{\frac{1}{4}})^4$ 45. $\left(\frac{f^{-16}}{256g^4h^{-4}}\right)^{-\frac{1}{4}}$ 46. $\sqrt[6]{x^2(x^{\frac{3}{4}} + x^{-\frac{3}{4}})}$ 47. $(2x^{\frac{1}{4}}y^{\frac{1}{3}})(3x^{\frac{1}{4}}y^{\frac{2}{3}})$

48. Show that $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$.

Express using rational exponents.

49. $\sqrt{m^6n}$ 50. $\sqrt{xy^3}$ 51. $\sqrt[3]{8x^3y^6}$

52. $17\sqrt[7]{x^{14}y^7z^{12}}$ 53. $\sqrt[5]{a^{10}b^2} \cdot \sqrt[4]{c^2}$ 54. $60\sqrt[8]{r^{80}s^{56}t^{27}}$

Express using a radical.

55. $16^{\frac{1}{5}}$ 56. $(7a)^{\frac{5}{8}}b^{\frac{3}{8}}$ 57. $p^{\frac{2}{3}}q^{\frac{1}{2}}r^{\frac{1}{3}}$

58. $\frac{2^{\frac{2}{3}}}{2^{\frac{1}{3}}}$ 59. $13a^{\frac{1}{7}}b^{\frac{1}{3}}$ 60. $(n^3m^9)^{\frac{1}{2}}$

61. What is the value of x in the equation $x = \sqrt[3]{(-245)^{-\frac{1}{5}}}$ to the nearest hundredth?

Simplify each expression.

62. $\sqrt{d^3e^2f^2}$ 63. $\sqrt[3]{a^5b^7c}$ 64. $\sqrt{20x^3y^6}$

Solve each equation.

65. $14.2 = x^{-\frac{3}{2}}$ 66. $724 = 15a^{\frac{5}{2}} + 12$ 67. $\frac{1}{8}\sqrt{x^5} = 3.5$

Warm Up

32. Simplify $4x^2(4x)^{-2}$. *(Lesson 11-1)*

36. **Sports** Suppose a baseball player popped a baseball straight up at an initial velocity v_0 of 72 feet per second. Its distance s above the ground after t seconds is described by $s = v_0t - 16t^2 + 4$. Find the maximum height of the ball. *(Lesson 10-5)*

38. Verify that $\sin^4 A + \cos^2 A = \cos^4 A + \sin^2 A$ is an identity. *(Lesson 7-2)*

39. **Mechanics** A circular saw 18.4 centimeters in diameter rotates at 2400 revolutions per second. What is the linear velocity at which a saw tooth strikes the cutting surface in centimeters per second? *(Lesson 6-2)*

40. **Travel** Martina went to Acapulco, Mexico, on a vacation with her parents. One of the sights they visited was a cliff-diving exhibition into the waters of the Gulf of Mexico. Martina stood at a lookout site on top of a 200-foot cliff. A team of medical experts was in a boat below in case of an accident. The angle of depression to the boat from the top of the cliff was 21°. How far is the boat from the base of the cliff? *(Lesson 5-4)*

41. **Salary** Diane has had a part time job as a Home Chef demonstrator for 9 years. Her yearly income is listed in the table. Write a model that relates the income as a function of the number of years since 1990. *(Lesson 4-8)*

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998
Income ($)	4012	6250	7391	8102	8993	9714	10,536	11,362	12,429

Example 1: Graph the exponential functions $y = 4^x$, $y = 4^x + 2$, $y = 4^x - 3$ on the same set of axes. Compare and contrast the graphs.

Example 2: Graph the exponential functions $y = \left(\frac{1}{5}\right)^x$, $y = 6\left(\frac{1}{5}\right)^x$, $y = -2\left(\frac{1}{5}\right)^x$ on the same set of axes. Compare and contrast the graphs.

Example 3: According to Newton's Law of Cooling, the difference between the temperature of an object and its surroundings decreases in time exponentially. Suppose a certain cup of coffee is 95° C and it is in a room that is 20° C. The cooling for this particular cup can be modeled by the equation $y = 75(0.875)^t$ where y is the temperature difference and t is the time in minutes.

(a) Find the temperature of the coffee after 15 minutes.

(b) Graph the cooling function

(c) Using a graphing calculator, determine how long it will take for the coffee to cool about 15° C.

Exponential Growth or Decay	The equation $N = N_0(1 + r)^t$, where N is the final amount, N_0 is the initial amount, r is the rate of growth or decay per time period, and t is the number of time periods, is used for modeling exponential growth or decay.

Example 4: Suppose that a researcher estimates that the initial population of honeybees in a colony is 500. They are increasing at a rate of 14% per week.

(a) What is the expected population in 22 weeks?

(b) When will the population reach 5000 honeybees?

Compound Interest	The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is paid, or compounded each year, and t is the number of years.

Example 5: Determine the amount of money in a money market account providing an annual rate of 5% compounded daily if a person invested $2000 and left it in the account for 7 years.

Class work

24. **Employment** Average national teachers' salaries can be modeled using the equation $y = 9.25(1.06)^n$, where y is the salary in thousands of dollars and n is the number of years since 1970.
    a. Graph the function.
    b. Using this model, what can a teacher expect to have as a salary in the year 2020?

25. **Aviation** When kerosene is purified to make jet fuel, pollutants are removed by passing the kerosene through a special clay filter. Suppose a filter is fitted in a pipe so that 15% of the impurities are removed for every foot that the kerosene travels.
    a. Write an exponential function to model the percent of impurity left after the kerosene travels x feet.
    b. Graph the function.
    c. About what percent of the impurity remains after the kerosene travels 12 feet?
    d. Will the impurities ever be completely removed? Explain.

26. **Demographics** Find the projected population of each location in 2015.
    a. In Honolulu, Hawaii, the population was 836,231 in 1990. The average yearly rate of growth is 0.7%.
    b. The population in Kings County, New York has demonstrated an average decrease of 0.45% over several years. The population in 1997 was 2,240,384.

27. **Biology** Scientists who study Atlantic salmon have found that the oxygen consumption of a yearling salmon O is given by the function $O = 100\left(3^{\frac{3s}{5}}\right)$, where s is the speed that the fish is traveling in feet per second.
    a. What is the oxygen consumption of a fish that is traveling at 5 feet per second?

    b. If a fish has traveled 4.2 miles in an hour, what is its oxygen consumption?

Warm Up

1) A model relating the average number of crimes reported at a shopping mall as a function of the number of years since 1991 is $y = -1.5x^2 + 13.3x + 19.4$. According to this model, what is the number of crimes for the year 2001?

Notes

Recall the Compound Interest Equation: $A = P(1+\frac{r}{n})^{nt}$. Suppose someone invested $1 at a bank which pays an interest rate of 100%. Suppose the person leaves the money in the account for 1 year. We know that the more frequently the money is compounded (how often the interest is added) then the more money is accumulated. Do you think that if the number of frequencies it is compounded increases causes the amount of money earned to increase forever?

Using $A = 1(1+\frac{1}{n})^{n}$, complete the t-table below for the values of n

n	A
12 (monthly)	
52 (weekly)	
360 (daily)	
8640 (hourly)	
31104000 (every second)	

The value A approaches an irrational number we symbolize as "e" $e \approx 2.71$

Exponential Growth or Decay (in terms of *e*)	$N = N_0e^{kt}$, where N is the final amount, N_0 is the initial amount, k is a constant and t is time.

Example 1: Assume that there is 1,000,000 kilograms of a radioactive substance in a city in 1973 and the decay constant of this substance is k = -0.0211.

(a) Write a function to model the amount of the substance remaining in the city.

(b) Find the amount of the substance that will be in the city in 2005.

(c) Graph the function and use the graph to find how long it will take for only 1,000 kilograms of the substance to remain.

Continuously Compounded Interest	The equation $A = Pe^{rt}$, where P is the initial amount, A is the final amount, r is the annual interest rate, and t is time in years, is used for calculating interest that is compounded continuously.

Example 2: Compare the balance after 25 years of a $10,000 investment earning 6.75% interest compounded continuously to the same investment compounded semiannually.

Class work

6. **Demographics** Bakersfield, California was founded in 1859 when Colonel Thomas Baker planted ten acres of alfalfa for travelers going from Visalia to Los Angeles to feed their animals. The city's population can be modeled by the equation $y = 33{,}430e^{0.0397t}$, where t is the number of years since 1950.
   a. Has Bakersfield experienced growth or decline in population?
   b. What was Bakersfield's population in 1950?
   c. Find the projected population of Bakersfield in 2010.

7. **Financial Planning** The Kwans are saving for their daughter's college education. If they deposit $12,000 in an account bearing 6.4% interest compounded continuously, how much will be in the account when Ann goes to college in 12 years?

8. **Psychology** Without further study, as time passes you forget things you have learned. The Ebbinghaus model of human memory gives the percent p of acquired knowledge that a person retains after t weeks. The formula is $p = (100 - a)e^{-bt} + a$, where a and b vary from one person to another. If $a = 18$ and $b = 0.6$ for a certain student, how much information will the student retain two weeks after learning a new topic?

9. **Physics** Newton's Law of Cooling expresses the relationship between the temperature of a cooling object y and the time t elapsed since cooling began. This relationship is given by $y = ae^{-kt} + c$, where c is the temperature of the medium surrounding the cooling object, a is the difference between the initial temperature of the object and the surrounding temperature, and k is a constant related to the cooling object.
   a. The initial temperature of a liquid is 160°F. When it is removed from the heat, the temperature in the room is 76°F. For this object, $k = 0.23$. Find the temperature of the liquid after 15 minutes.
   b. Alex likes his coffee at a temperature of 135°. If he pours a cup of 170°F coffee in a 72°F room and waits 5 minutes before drinking, will his coffee be too hot or too cold? Explain. For Alex's cup, $k = 0.34$.

12. **Sociology** Sociologists have found that information spreads among a population at an exponential rate. Suppose that the function $y = 525(1 - e^{-0.038t})$ models the number of people in a town of 525 people who have heard news within t hours of its distribution.
   a. How many people will have heard about the opening of a new grocery store within 24 hours of the announcement?
   b. Graph the function on a graphing calculator. When will 90% of the people have heard about the grocery store opening?

13. **Customer Service** The service-time distribution describes the probability P that the service time of the customer will be no more than t hours. If m is the mean number of customers serviced in an hour, then $P = 1 - e^{-mt}$.
   a. Suppose a computer technical support representative can answer calls from 6 customers in an hour. What is the probability that a customer will be on hold less than 30 minutes?
   b. A credit card customer service department averages 34 calls per hour. Use a graphing calculator to determine the amount of time after which it is 50% likely that a customer has been served?

Common Logarithm – The common logarithm of a positive number x, denoted $\log x$, is defined by $\log x = k$ if and only if $x = 10^k$, where k is a real number. The function given by $f(x) = \log x$ is called the common logarithmic function.

Example 1: Simplify each logarithm by hand.

(a) $\log 100,000$ (b) $\log 1$ (c) $\log \frac{1}{1000}$ (d) $\log \sqrt{10}$ (e) $\log(-2)$

Inverse Properties of the Common Logarithm – The following inverse properties hold for the common logarithm. $\log 10^x = x$ for any real number x and $10^{\log x} = x$ for any positive number x.

Example 2: The number of malaria deaths in millions x years after the year 2000 can be modeled by $D(x) = 0.9\log(10 - \frac{2}{5}x)$.

(a) Evaluate D(0) by hand and interpret the result. (b) Approximate D(15) and interpret the result.

Logarithm – The logarithm with base a of a positive number x, denoted by $\log_a x$, is defined by $\log_a x = k$ if and only if $x = a^k$, where a > 0, $a \neq 1$, and k is a real number. The function, given by $f(x) = \log_a x$ is called the logarithmic function with base a.

Example 3: Evaluate each logarithm

(a) $\log_2 8$ (b) $\log_5 \frac{1}{25}$ (c) $\log_7 49$ (d) $\ln e^{-7}$

Inverse Properties – The following inverse properties hold for logarithms with base a. $\log_a a^x = x$ for any real number x and $a^{\log_a x} = x$ for any positive number x.

Example 4: Use inverse properties to evaluate each expression.

(a) $\log_6 6^{-1.3}$ (b) $5^{\log_5(x+8)}$ (c) $\log_{1/2}(\frac{1}{2})^{45}$

Example 5: Explain how to obtain the graph of $g(x) = \log(-x) + 1$ from the graph of $f(x) = \log x$. Graph both functions in the same xy-plane.

Example 6: Solve each equation, if possible.

(a) $10^x = 0.001$ (b) $10^x = 55$ (c) $10^x = -1$

Example 7: Solve $4(10^{3x}) = 244$

Example 8: Solve each equation

(a) $\log x = 3$ (b) $\log x = -2$ (c) $\log x = 2.7$

Example 9: Solve $5\log 2x = 16$

Example 10: Solve each equation

(a) $3^x = \frac{1}{27}$ (b) $e^x = 5$ (c) $3(2^x) - 7 = 20$

Example 11: Solve each equation

(a) $\log_2 x = 5$ (b) $\log_5 x = -2$ (c) $\ln x = 4.3$ (d) $3\log_2 5x = 9$

Class work

Write each equation in exponential form.

20. $\log_{27} 3 = \frac{1}{3}$
21. $\log_{16} 4 = \frac{1}{2}$
22. $\log_7 \frac{1}{2401} = -4$
23. $\log_4 32 = \frac{5}{2}$
24. $\log_e 65.98 = x$
25. $\log_{\sqrt{6}} 36 = 4$

Write each equation in logarithmic form.

26. $81^{\frac{1}{2}} = 9$
27. $36^{\frac{3}{2}} = 216$
28. $\left(\frac{1}{8}\right)^{-3} = 512$
29. $6^{-2} = \frac{1}{36}$
30. $16^0 = 1$
31. $x^{1.238} = 14.36$

Evaluate each expression.

32. $\log_8 64$
33. $\log_{125} 5$
34. $\log_2 32$
35. $\log_4 128$
36. $\log_9 9^6$
37. $\log_{49} 343$
38. $\log_8 16$
39. $\log_{\sqrt{8}} 4096$

Properties of Logarithms – For positive numbers m, n, and $a \neq 1$ and any real number r:

(1) $\log_a 1 = 0$ and $\log_a a = 1$

(2) $\log_a m + \log_a n = \log_a (mn)$

(3) $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$

(4) $\log_a (m^r) = r \log_a m$

Example 1: Use a calculator to evaluate each pair of expressions. Then state which rule of logarithms this calculator illustrates.

(a) $\ln 5 + \ln 4, \quad \ln 20$

(b) $\log 10 - \log 5, \quad \log 2$

(c) $\log 5^2, \quad 2\log 5$

Example 2: Use properties of logarithms to expand each expression. Write your answers without exponents.

(a) $\log xy$

(b) $\ln \frac{5}{z}$

(c) $\log_4 \frac{\sqrt[3]{x}}{\sqrt{k}}$

Example 3: Expand each expression. Write your answer without exponents.

(a) $\log_2 2x^4$

(b) $\ln \frac{7x^3}{k^2}$

(c) $\log \frac{\sqrt{x+1}}{(x-2)^3}$

Example 4: Write each expression as the logarithm of a single expression.

(a) $\ln 2e + \ln \frac{1}{e}$

(b) $\log_2 27 + \log_2 x^3$

(c) $\log x^3 - \log x^2$

Example 5: Write each expression as the logarithm of a single expression.

(a) $\log 5 + \log 15 - \log 3$

(b) $2\ln x - \frac{1}{2}\ln y - 3\ln z$

(c) $5\log_3 x + \log_3 2x - \log_3 y$

Change of Base Formula – Let x, $a \neq 1$, and $b \neq 1$ be positive real numbers. Then $\log_a x = \dfrac{\log_b x}{\log_b a}$.

Example 6: Use a calculator to approximate each expression to the nearest thousandth.

(a) $\log_4 20$ (b) $\log_2 125 + \log_7 39$

Example 7: Solve the equation $\log_4(2x+11) = \log_4(5x-4)$

Example 8: Solve the equation $\log_{11} x + \log_{11}(x+1) = \log_{11} 6$

Example 9: Solve the equation $6^{3x} = 81$

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Practice Test 8

Date________________ Period____

Divide.

1) $(r^3 - 43 - 3r^2 - 66r) \div (r - 10)$

Factor each completely.

2) $32p^3 - 20p^2 - 8p + 5$

Find all zeros.

3) $f(x) = 3x^3 - 23x^2 + 33x + 35$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

4) $-4,\ 3i,\ -3i$

Use the given point on the terminal side of angle θ to find the value of the trigonometric function indicated.

5) $\sec\theta;\ (5, \sqrt{11})$

Using radians, find the period of each function.

6) $y = \frac{1}{9} \cdot \sin\left(3\theta - \frac{3\pi}{4}\right)$

Perform the indicated operation.

7) $h(x) = x^2 - x$
$g(x) = 4x - 3$
Find $h(g(x))$

Find the inverse of the function.

8) $g(x) = -2x - 2$

Expand each logarithm.

9) $\log_6 (x \cdot y \cdot z^3)$

10) $\log_2 \sqrt{x \cdot y \cdot z}$

Condense each expression to a single logarithm.

11) $3\log_9 a + 3\log_9 c - 15\log_9 b$

12) $\log_7 d + \frac{\log_7 a}{2} + \frac{\log_7 b}{2} + \frac{\log_7 c}{2}$

Solve each equation.

13) $\log_{11}(3m - 8) = \log_{11}(2m + 2)$

14) $\log_7(2 - 7m) = 2$

15) $\log_2 6 + \log_2(x + 8) = 5$

16) $\log_3(x - 7) - \log_3 2 = \log_3 79$

Solve each equation. Round your answers to the nearest ten-thousandth.

17) $10^{2n-6} + 9 = 53$

18) $e^{2k+9} - 5 = 58$

19) The formula for finding the amount of an investment with interest compounded yearly for t years is: $A = P(1 + r)^t$, where A is the amount accrued, P is the principal, and r is the rate. James invests $2,345 at a rate of 5.13% interest compounded yearly. Approximately how many years will he have to invest his money in order to have $4,000? Round your answer to the nearest year.

20) The altitude of an aircraft is in part affected by the outside air pressure and can be determined by the equation $h = -\frac{100}{9}\log\frac{P}{B}$, where "h" is the altitude in miles, "P" is the air pressure outside the aircraft, and "B" is the air pressure at sea level. Normally B = 14.7 pounds per square inch. Suppose the air pressure outside an airplane is 11.3. What is the altitude of the plane?

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Test 8 - Form A
Date________________ Period____

Divide.

1) $\left(7x^2 + x^3 + 17x + 11\right) \div (3 + x)$

Factor each completely.

2) $5n^3 - 8n^2 - 20n + 32$

Find all zeros.

3) $f(x) = x^3 + 2x + 12$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

4) $5, \ i, \ -i$

Use the given point on the terminal side of angle θ to find the exact value for theta.

5) $\sec\theta$; $\left(7, \sqrt{15}\right)$

Using radians, find the period for the function.

6) $y = \frac{1}{8} \cdot \cos\left(4\theta - \frac{\pi}{4}\right)$

Perform the indicated operation.

7) $g(x) = x^2 - 2x$
$f(x) = -2x - 4$
Find $g(f(x))$

Find the inverse of the function.

8) $f(x) = 5x + 10$

Expand each logarithm.

9) $\log_6 \sqrt[3]{x \cdot y \cdot z}$

10) $\log_2 (a^4 b^3)$

Condense each expression to a single logarithm.

11) $\log_4 w + \dfrac{\log_4 x}{2} + \dfrac{\log_4 y}{2} + \dfrac{\log_4 z}{2}$

12) $\log_9 w + 4\log_9 u - 4\log_9 v$

Solve each equation.

13) $\log_5 (r+9) = \log_5 (3r-9)$

14) $\log_2 (10x+8) = 2$

15) $\log_3 (x-9) + \log_3 4 = \log_3 71$

16) $\log_8 (x+8) - \log_8 2 = 2$

Solve each equation. Round your answers to the nearest ten-thousandth.

17) $3 \cdot 10^{5-4p} = 9$

18) $e^{-10x-1} - 6 = 61.5$

19) The formula for finding the amount of an investment with interest compounded yearly for t years is: $A = P(1 + r)^t$, where A is the amount accrued, P is the principal, and r is the rate. James invests $2,345 at a rate of 5.13% interest compounded yearly. Approximately how many years will he have to invest his money in order to have $5,000? Round your answer to the nearest year.

20) The altitude of an aircraft is in part affected by the outside air pressure and can be determined by the equation $h = -\frac{100}{9}\log\frac{P}{B}$, where "h" is the altitude in miles, "P" is the air pressure outside the aircraft, and "B" is the air pressure at sea level. Normally B = 14.7 pounds per square inch. Suppose the air pressure outside an airplane is 10.3. What is the altitude of the plane?

Test 9 Day 1

Warm Up – The Difference Quotient

Given $f(x) = x^2$

Find $$\frac{f(x+h) - f(x)}{h}$$

1

Essential Question

- What are the fundamental concepts of Calculus?
- Vocabulary
  - Differential Calculus
  - Integral Calculus
- Quiz!
- Test!

2

1.1 A Preview of Calculus

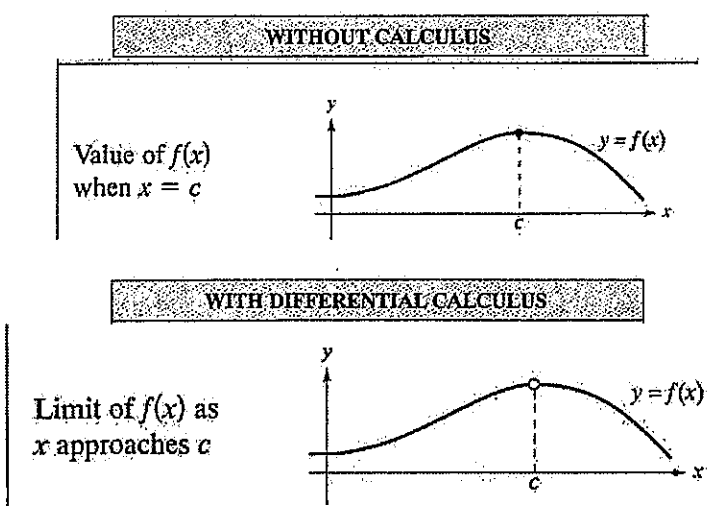
What is Calculus?

- Calculus is the mathematics of CHANGE
- Calculus is a more dynamic version of pre-calculus – more complex and applicable to real world situations – multidimensional
- Calculus is not a different math, it is a continuation of what you have already learned with applications to actual real world situations

3

1.1 A Preview of Calculus

Example 1



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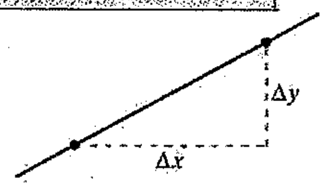
1.1 A Preview of Calculus

Example 2

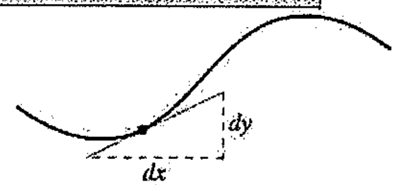
WITHOUT CALCULUS

Slope of a line



WITH DIFFERENTIAL CALCULUS

Slope of a curve



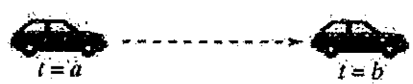
5

1.1 A Preview of Calculus

Example 3

WITHOUT CALCULUS

Average rate of change between $t = a$ and $t = b$



WITH DIFFERENTIAL CALCULUS

Instantaneous rate of change at $t = c$

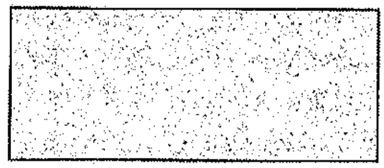
6

1.1 A Preview of Calculus

Example 4

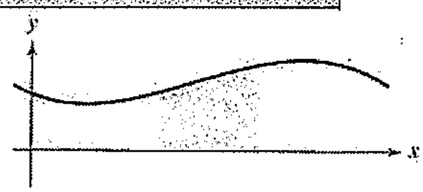
WITHOUT CALCULUS

Area of a rectangle



WITH DIFFERENTIAL CALCULUS

Area under a curve



7

1.1 A Preview of Calculus

Example 5

WITHOUT CALCULUS

Surface area of a cylinder

WITH DIFFERENTIAL CALCULUS

Surface area of a solid of revolution

8

1.1 A Preview of Calculus

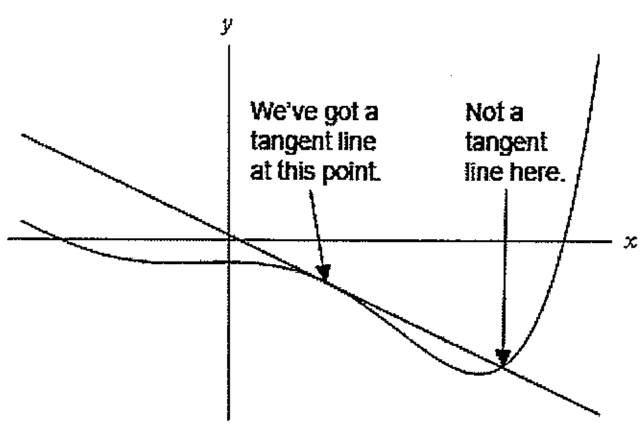
The 3 Main Ideas

- Limits – detailed curve sketching
- Derivatives – rates of change/slope of tangent lines
- Anti-Derivatives – area and volume of any shape or solid

9

1.1 A Preview of Calculus

Tangent Line Problem



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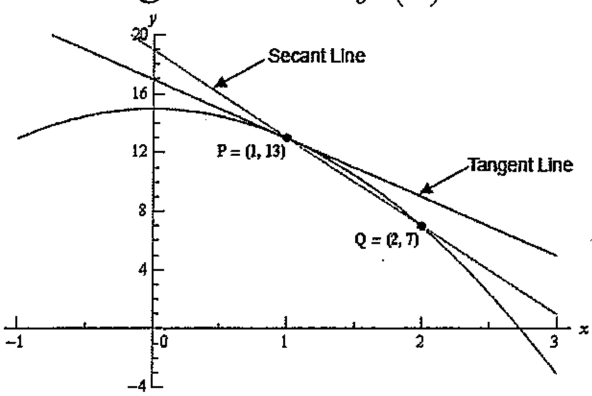
1.1 A Preview of Calculus

1. Find the tangent line to $f(x) = 15 - 2x^2$ at $x = 1$.

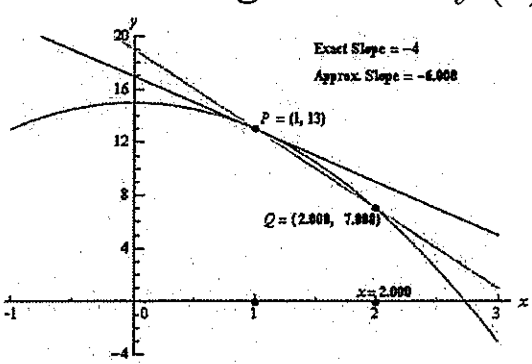


$$m_{PQ} = \frac{f(2) - f(1)}{2 - 1} = \frac{7 - 13}{1} = -6$$

11

1.1 A Preview of Calculus

1. Find the tangent line to $f(x) = 15 - 2x^2$ at $x = 1$.



x	m_{PQ}
2	−6
1.5	−5
1.1	−4.2
1.01	−4.02
1.001	−4.002
1.0001	−4.0002

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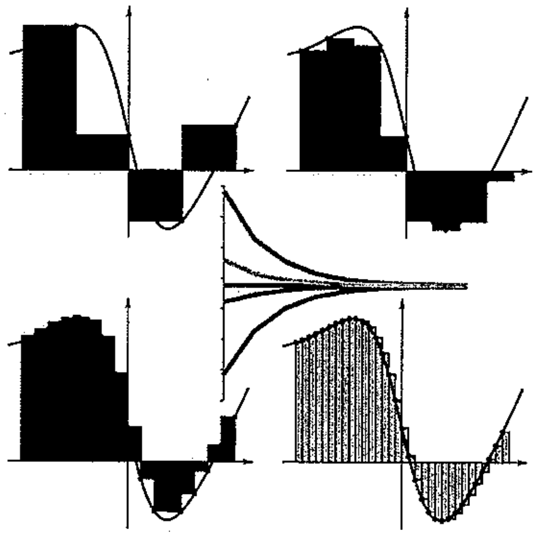
1.1 A Preview of Calculus

Area Problem



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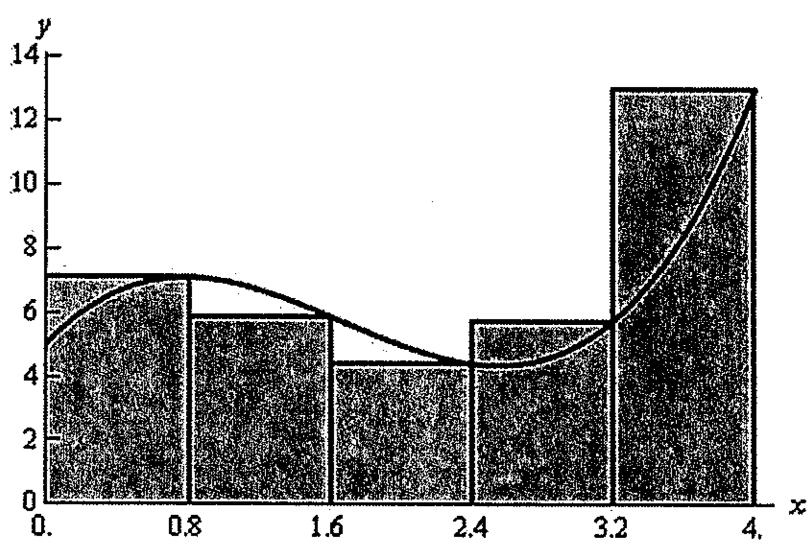
1.1 A Preview of Calculus



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1.1	A Preview of Calculus

Class work

Pg. 47, 1.1 #1-9

15

1.1	A Preview of Calculus

Homework

Watch the video

AP Calculus Lesson 1.2

16

Test 9 - Day 2

Warm Up

Section 1.2 – Limits: A numerical and Graphical Approach

1

Example 1

Investigate the function numerically and graphically

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$$

2

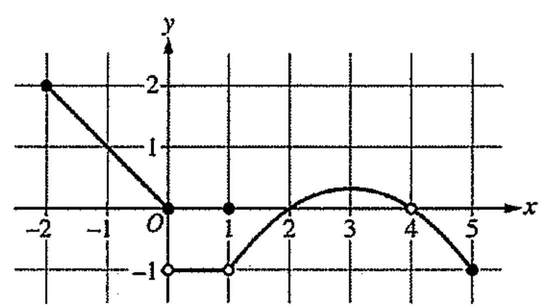
Example 2

Investigate the function numerically and graphically

$$\lim_{h \to 0} \frac{e^h - 1}{h}$$

3

Example 3

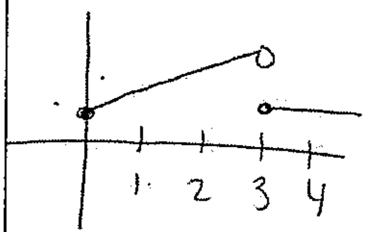


Graph of f

The graph of the function f is shown above. For what values of a does $\lim_{x \to a} f(x) = 0$?

4

Example 4



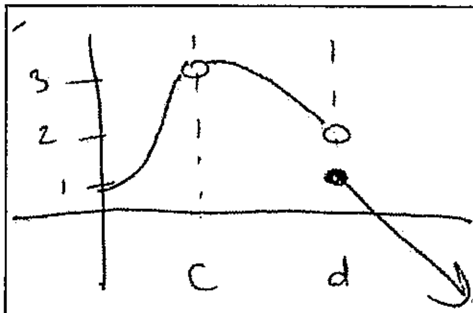
$\lim_{x \to 3^-} f(x)$

$\lim_{x \to 3^+} f(x)$

$\lim_{x \to 3} f(x)$

5

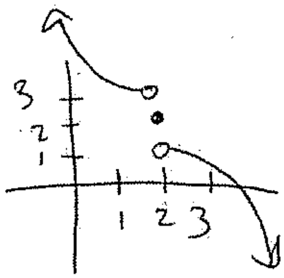
Example 5



$\lim_{x \to c} g(x)$

$\lim_{x \to d} g(x)$

6



Example 6

$\lim_{x \to 2^-} f(x)$

$\lim_{x \to 2^+} f(x)$

$\lim_{x \to 2}$

7

Test 9 – Day 3

[redacted]

[redacted]

1

[redacted]

Scalar Multiple, Sum or difference, Product, Quotient, Power

2

Direct substitution is a valid analytical method to evaluate the following limits.

- If p is a **polynomial function** and c is a real number, then:

$$\lim_{x \to c} p(x) = p(c)$$

- If r is a **rational function** given by $r(x) = p(x)/q(x)$, and c is a real number, then

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{q(c)}, \quad q(c) \neq 0$$

- If a **radical function** where n is a positive integer. The following limit is valid for all c if n is odd and only $c>0$ when n is even:

$$\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$$

3

Direct substitution is a valid analytical method to evaluate the following limits.

- If the f and g are functions such that $\lim_{x \to c} g(x) = L$ *and* $\lim_{x \to L} f(x) = f(L)$ Then the limit of the **composition** is:

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L)$$

- If c is a real number in the domain of a **trigonometric function** then:

$$\lim_{x \to c} \sin x = \sin c \qquad \lim_{x \to c} \cos x = \cos c$$

$$\lim_{x \to c} \tan x = \tan c \qquad \lim_{x \to c} \cot x = \cot c$$

$$\lim_{x \to c} \sec x = \sec c \qquad \lim_{x \to c} \csc x = \csc c$$

4

Evaluate the limit analytically:

$$\lim_{x\to 2}\frac{x^2-4x+4}{x^2-x-2}$$

5

To find limits analytically, try the following:

1. Direct Substitution (Try this FIRST)

2. If Direct Substitution fails, then rewrite then find a function that is equivalent to the original function except at one point. Then use Direct Substitution. Methods for this include…

   - Factoring/Dividing Out Technique
   - Rationalize Numerator/Denominator
   - Eliminating Embedded Denominators
   - Trigonometric Identities
   - Legal Creativity

6

Evaluate the limit analytically:

$$\lim_{x\to 2}\frac{x^2-4}{x^2-x-2}$$

7

Evaluate the limit analytically:

$$\lim_{y\to 2}\frac{\sqrt{y+2}-2}{y-2}$$

8

Evaluate the limit analytically:

$$\lim_{h\to 0}\frac{3h^2+2h}{h}$$

9

The following limits can be assumed to be true to assist in finding other limits:

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

10

[redacted]

Evaluate the limit analytically:

$$\lim_{x \to 0} \frac{\sin 3x}{5x}$$

11

[redacted]

Evaluate the limit analytically:

$$\lim_{x \to 0} \frac{1-\cos^2(2x)}{x^2}$$

12

Evaluate the limit analytically:

$$\lim_{x \to 0} \frac{\sin x \cos x}{x}$$

13

Evaluate the limit analytically:

$$\lim_{x \to 0} (x \csc x)$$

14

[redacted]

Evaluate the limit analytically:

$$\lim_{x\to 0}\frac{1-\cos x}{2\sin^2 x}$$

15

[redacted]

16

Lesson 2.1 Test 9 – Day 4

Warm Up

1. In your own words, describe what the derivative is in geometric terms.
2. What is the formula for the definition of the derivative of a function?
3. What is the derivative of $f(x) = 10x - 2$
4. If a function is continuous at a point, does that imply it is differentiable at that point also? Give an example.

Notes – Write on back on this paper.

Class Work – Do on separate sheet of paper and staple to this paper.

P103 #1-4, 5, 7, 11, 13, 15, 17, 37-40, 43-46, 65, 81-86

TOD

1. Find $f'(2)$ for the function $f(x) = 3x^2 - 4x$

Lesson 2.2

Test 9 – Day 5

Warm Up

1. What is the derivative of any constant?

2. What is the derivative of e^x and $\ln x$?

3. What is the derivative of $\sin x$ and $\cos x$?

4. Find the derivative of the following function: $f(t) = -2t^2 + \dfrac{4}{5t^3} + \cos t + \ln t - e^t - \sqrt[4]{t^3} + \pi$

Notes – Write on back on this paper.

Class work – Do on separate sheet of paper and staple to this paper.

P115 #3-27 (odd), #39-51 (odd), 89, 113, 114

TOD

1. Find an equation of the tangent line to the graph of f at the given point.

$$f(x) = \frac{2}{\sqrt[4]{x^3}} \qquad (1, 2)$$

Lesson 2.3 Product Rule

Test 9 - Day 6

Warm Up

1. Evaluate $\lim_{h \to 0} \frac{\cos(\pi + h) - \cos(\pi)}{h}$

2. Using a graphing calculator, find all points where the function $f(x) = x^{2/3} + 1$ is not differentiable.

3. Using a calculator, find the derivative of $f(x) = x^2 + 4x$ at x = 3.

4. Find $f'(3)$ if $f(x) = \frac{1}{\sqrt{x}}$

5. Find $\frac{dy}{dx}$ if $y = (3x-5)(x^4 + 8x - 1)$

Notes – Write on back on this paper.

Class work – Do on separate sheet of paper and staple to this paper.

P126 #1-6, 13, 14, 17, 30, 39, 50, 52

TOD

1. Find the inverse of the function $f(x) = x^3 + 1$

Lesson 2.3 Quotient Rule

Test 9 – Day 7

Warm Up

1. If $f(x)=\frac{2x-1}{x+5}$, find $f'(x)$

2. Find an equation of the tangent to the curve $f(x)=x^2-3x+2$ at x = 5.

Notes – Write on back on this paper.

Class work – Do on separate sheet of paper and staple to this paper.

P126 #7-12, 15, 16, 19-24, 41, 42

TOD

1. Let $f(x)=\frac{1}{x+5}$ and $g(x)=x^2$. Find $f(g(3))$

Lesson 2.4 Introduction to Chain Rule

Test 9 - Day 8

Warm Up

1. If $y=(3x-5)^{10}$, find $\frac{dy}{dx}$

2. If $f(x)=5x\sqrt{25-x^2}$, find $f'(x)$

3. If $y=\left(\frac{2x-1}{x^2}\right)^3$, find $\frac{dy}{dx}$

Notes – Write on back on this paper.

Class work – Do on separate sheet of paper and staple to this paper.

P137 #1-3, 7, 13, 14, 59, 60, 67, 70

TOD

1. Evaluate $\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}\right)}{h}$

Acc PreCalc

Name________________________________ ID: 1

Test 9 Form A

Date________________ Period____

1) $\lim_{h \to 25} \frac{\sqrt{h} - 5}{h - 25}$

2) $\lim_{n \to -\infty} \frac{3n^3 - 5n}{n^3 - 2n^2}$

3) $\lim_{x \to 0} \frac{5x^4 + 8x^2}{3x^4 - 16x^2}$

4) $\lim_{x \to -\infty} \frac{(2x - 1)(3 - x)}{(x - 1)(x + 3)}$

5) $\lim_{x \to 0} \frac{2x^6 + 6x^3}{4x^5 + 3x^3}$

6) $\lim_{x \to -1} \frac{x^2 - 3x - 4}{x^2 - 1}$

Solve each equation. Round your answers to the nearest ten-thousandth.

7) $\log_3 (5x + 2) - \log_3 2 = 4$

8) $-8e^{6n - 1} + 5.8 = -90$

Find all zeros.

9) $f(x) = 5x^4 - 46x^2 + 9$

Approximate the relative minima and relative maxima of each function to the nearest tenth.

10) $f(x) = x^4 - x^3 - 3x^2 + 3$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

11) $-2,\ -i,\ i$

For #12-15, Find the derivative of the following functions.

12) $f(x) = \sin(2x)e^{3x}$

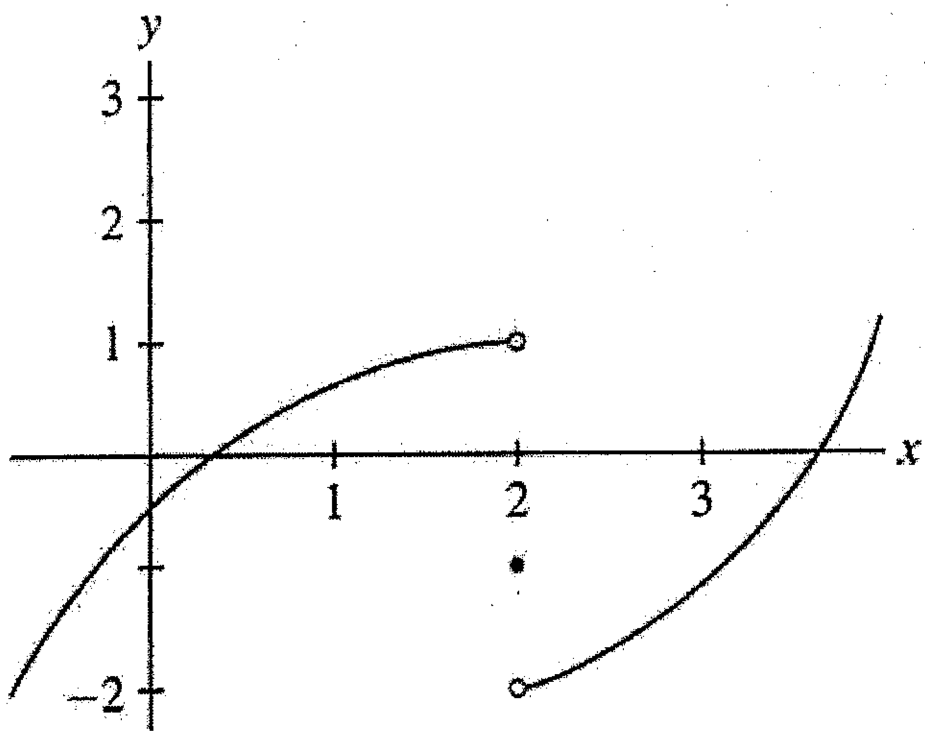
13) $f(x) = \sin x \cdot x^2$

14) $f(x) = -2x^3 + 5x^2 + \ln x + e^x + \sin x$

15) $f(x) = \dfrac{\cos x}{x^3}$

16) The distance between Miami and Orlando is about 200 miles. A pilot flying from Miami to Orlando starts the flight 12° off course to avoid a storm. After flying in this direction for 80 miles, how far is the plane from Orlando?

17)



Which of the following statements below are true? Circle all that apply.

I. $\lim_{x \to 2^-} f(x) = 1$

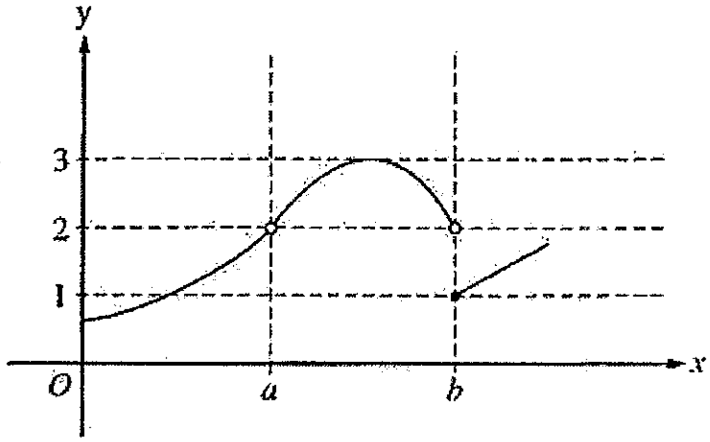
II. $\lim_{x \to 2^+} f(x) = 1$

III. $\lim_{x \to 2^-} f(x) = -1$

IV. $\lim_{x \to 2^+} f(x) = -1$

V. $\lim_{x \to 2} f(x) = 0$

18)



The graph of the function f is shown in the figure above. Which of the following statements about f is true?

a. $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

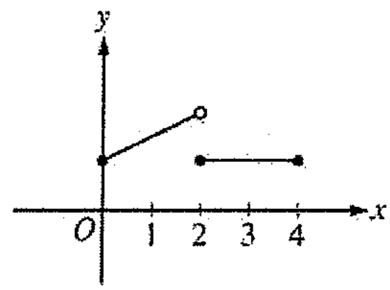
b. $\lim_{x \to b} f(x) = 2$

c. $\lim_{x \to a} f(x) = DNE$

d. $\lim_{x \to a} f(x) = 2$

e. $\lim_{x \to b} f(x) = 1$

19)



Graph of f

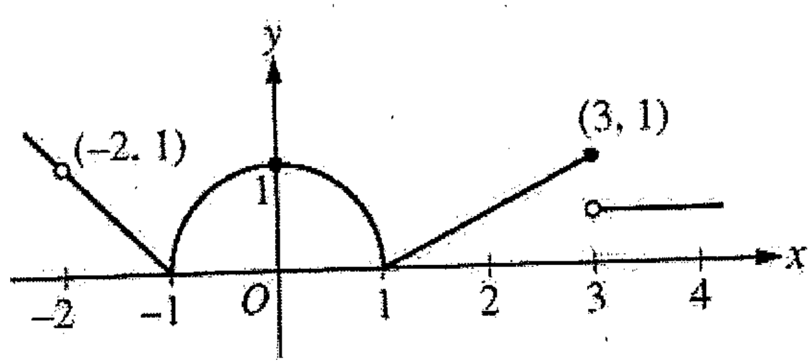
The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x)$ exists.

II. $\lim_{x \to 2^+} f(x)$ exists.

III. $\lim_{x \to 2} f(x)$ exists.

20)



Graph of f

The graph of a function f is shown above. For what value(s) of c does $\lim_{x \to c} f(x) = 1$?