Algebra 2

Packet #2

Topics Covered:

Test 6 – Rational Functions

Test 7 – Exponential and Logarithmic Functions

This packet consists of daily Notes with examples and practice problems. Solutions to these worksheets are found on "Schoology." There are 2 Tests in this packet. You are to take these 2 tests and submit for grading. Your scores on these 2 tests will determine the number of awarded points (out of 5 max points).

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Warm-Up

1) Approximate the real zeros of $g(x)=x^4-9x^3+25x^2-24x+6$ to the nearest tenth.

Notes

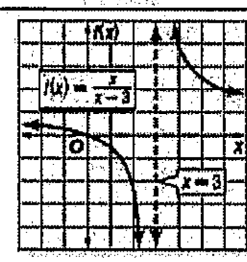
VERTICAL ASYMPTOTES AND POINT DISCONTINUITY The function $c=\frac{150}{s}$ is an example of a rational function. A **rational function** is an equation of the form $f(x)=\frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x)\neq 0$. Here are other examples of rational functions.

$$f(x)=\frac{x}{x+3} \qquad g(x)=\frac{5}{x-6} \qquad h(x)=\frac{x+4}{(x-1)(x+4)}$$

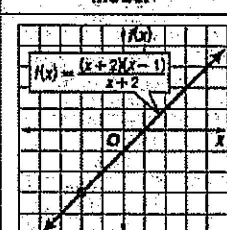
No denominator in a rational function can be zero because division by zero is not defined. In the examples above, the functions are not defined at $x=-3$, $x=6$, and $x=1$ and $x=-4$, respectively.

The graphs of rational functions may have breaks in **continuity**. This means that, unlike polynomial functions, which can be traced with a pencil never leaving the paper, not all rational functions are traceable. Breaks in continuity can appear as a vertical **asymptote** or as a **point discontinuity**. Recall that an asymptote is a line that the graph of the function approaches, but never crosses. Point discontinuity is like a hole in a graph.

Key Concept *Vertical Asymptotes*

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x=a$, then $x=a$ is a vertical asymptote.	For $f(x)=\frac{x}{x-3}$, $x=3$ is a vertical asymptote.	

Key Concept *Point Discontinuity*

Property	Words	Example	Model
Point Discontinuity	If the original function is undefined for $x=a$ but the rational expression of the function in simplest form is defined for $x=a$, then there is a hole in the graph at $x=a$.	$f(x)=\frac{(x+2)(x-1)}{x+2}$ can be simplified to $f(x)=x-1$. So, $x=-2$ represents a hole in the graph.	

Example 1: Graph $f(x)=\frac{x}{x+1}$. State the domain and range. Find any vertical and horizontal asymptotes.

Example 2: Graph $f(x)=\frac{6}{(x-2)(x+3)}$. Find any vertical and horizontal asymptotes.

Example 3: Graph $g(x)=\frac{x^2-25}{x-5}$. Find any vertical and horizontal asymptotes.

Class work

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

16. $f(x)=\frac{2}{x^2-5x+6}$
17. $f(x)=\frac{4}{x^2+2x-8}$
18. $f(x)=\frac{x+3}{x^2+7x+12}$
19. $f(x)=\frac{x-5}{x^2-4x-5}$
20. $f(x)=\frac{x^2-8x+16}{x-4}$
21. $f(x)=\frac{x^2-3x+2}{x-1}$

Graph each rational function.

22. $f(x)=\frac{1}{x}$
23. $f(x)=\frac{3}{x}$
24. $f(x)=\frac{1}{x+2}$
25. $f(x)=\frac{-5}{x+1}$
26. $f(x)=\frac{x}{x-3}$
27. $f(x)=\frac{5x}{x+1}$
28. $f(x)=\frac{-3}{(x-2)^2}$
29. $f(x)=\frac{1}{(x+3)^2}$
30. $f(x)=\frac{x+4}{x-1}$
31. $f(x)=\frac{x-1}{x-3}$
32. $f(x)=\frac{x^2-36}{x+6}$
33. $f(x)=\frac{x^2-1}{x-1}$
34. $f(x)=\frac{3}{(x-1)(x+5)}$
35. $f(x)=\frac{-1}{(x+2)(x-3)}$
36. $f(x)=\frac{x}{x^2-1}$
37. $f(x)=\frac{x-1}{x^2-4}$
38. $f(x)=\frac{6}{(x-6)^2}$
39. $f(x)=\frac{1}{(x+2)^2}$

Warm Up

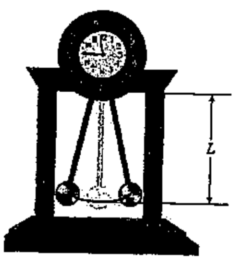
Find the vertical and horizontal asymptotes for the following function: $f(x)=\frac{x^2+1}{x+1}$

Notes

Direct Variation with the nth Power – Let x and y denote two quantities and n be a positive real number. Then y is directly proportional to the nth power of x, or y varies directly with the nth power of x, if there exists a nonzero number k such that $y=kx^n$. The number k is called the constant of variation or the constant of proportionality. In the formula $A=\pi r^2$, the constant of variation is π.

Example 1: If y varies directly as x and y = 9 when x is -15, find y when x = 21.

Example 2: The time T required for a pendulum to swing back and forth once is called its period. The length L of a pendulum is directly proportional to the square of T. A 2-foot pendulum has a 1.57-second period.



(a) Find the constant of proportionality k. Round to the nearest hundredth.

(b) Predict T for a pendulum having a length of 5 feet.

Inverse Variation with the nth Power – Let x and y denote two quantities and n be a positive real number. Then y is inversely proportional to the nth power of x, or y varies inversely with the nth power of x, if there exists a nonzero number k such that $y=\frac{k}{x^n}$. If $y=\frac{k}{x}$, then y is inversely proportional to x or y varies inversely with x.

Example 3: If y varies inversely as x and y = 4 when x = 12, find y when x = 5.

Example 4: Inverse variation occurs in measuring the intensity of light. If we increase our distance from a light bulb, the intensity of the light decreases. Intensity I is inversely proportional to the second power of the distance d. The equation $I=\frac{k}{d^2}$ models this phenomenon. At a distance of 3 meters, a 100-watt bulb produces an intensity of 0.88 watt per square meter.

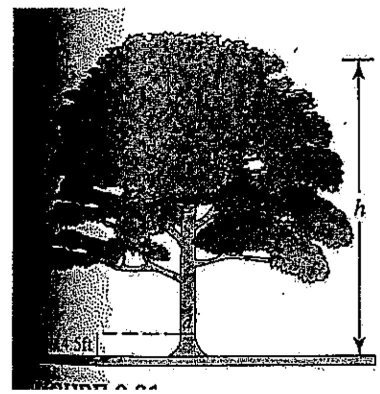
(a) Find the constant of proportionality k

(b) Determine the intensity at a distance of 2 meters.

Joint Variation – A quantity may depend on more than one variable. For example, the volume V of a cylinder is given by $V = \pi r^2 h$. We say that V varies jointly with h and the square of r. The constant of variation if π.

Let m and n be real numbers. Then z varies jointly with the mth power of x and the nth power of y if a nonzero real number k exists such that $z = kx^m y^n$

Example 5: To estimate the volume of timber in a given area of forest, formulas have been developed to find the amount of wood contained in a tree with height h in feet and diameter d in inches. See the figure below. One study concluded that the volume V of wood in a tree varies jointly with the 1.12 power of h and the 1.98 power of d. (The diameter is measured 4.5 feet above the ground).



(a) Write an equation that relates V, h, and d.

(b) A tree with a 13.8-inch diameter and a 64-foot height has a volume of 25.14 cubic feet. Estimate the constant of variation k.

(c) Estimate the volume of wood in a tree with d = 11 inches and h = 47 feet.

Check For Understanding

Find each value.

26. If y varies directly as x and $y = 15$ when $x = 3$, find y when $x = 12$.

27. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$.

28. Suppose y varies jointly as x and z. Find y when $x = 2$ and $z = 27$, if $y = 192$ when $x = 8$ and $z = 6$.

29. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$.

30. If y varies inversely as x and $y = 5$ when $x = 10$, find y when $x = 2$.

31. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$.

32. If y varies inversely as x and $y = 2$ when $x = 25$, find x when $y = 40$.

33. If y varies inversely as x and $y = 4$ when $x = 12$, find y when $x = 5$.

34. If y varies directly as x and $y = 9$ when x is -15, find y when $x = 21$.

35. If y varies directly as x and $x = 6$ when $y = 0.5$, find y when $x = 10$.

36. Suppose y varies jointly as x and z. Find y when $x = \frac{1}{2}$ and $z = 6$, if $y = 45$ when $x = 6$ and $z = 10$.

37. If y varies jointly as x and z and $y = \frac{1}{8}$ when $x = \frac{1}{2}$ and $z = 3$, find y when $x = 6$ and $z = \frac{1}{3}$.

Warm Up

1) Find any vertical and horizontal asymptotes for $f(x)=\dfrac{x+2}{x^2-4}$

2) Write the formulas for direct, inverse, and joint variation.

Notes

Example 1: Simplify $\dfrac{2x(x+1)}{(x+1)(x^2-4)}$

Example 2: Simplify $\dfrac{3a^3-a^4}{2a^3-6a^2}$

Example 3: Simplify each expression

(a) $\dfrac{2a^2}{5b^2c}\cdot\dfrac{3bc^2}{8a^3}$

(b) $\dfrac{8x^2y}{15a^2b}\div\dfrac{2xy^2}{5ab^4}$

Example 4: Find $\dfrac{x^2+2x-8}{x^2+4x+3}\div\dfrac{x-2}{3x+3}$

Example 5: Simplify $\dfrac{\dfrac{5a^2-20}{2a+2}}{\dfrac{10a-20}{4a}}$

Guided Practice p566 #8-15

Class work p566 #17, 19, 20, 21, 23, 24, 28,32, 38, 40

Warm Up

1) Simplify $\frac{m^3}{3n} \div \frac{m^4}{9n^2}$

2) Simplify $\frac{4a+4}{3} \cdot \frac{1}{a+1}$

Notes

Example 1: Simplify $\frac{2x}{5ab^3} + \frac{4y}{3a^2b^2}$

Example 2: Simplify $\frac{x}{x^2+5x+6} - \frac{2}{x^2+4x+4}$

Example 3: Simplify $\frac{x-5}{2x-6} - \frac{x-7}{4x-12}$

Guided Practice p573 #7-12

Class work p573 #20, 24, 27, 28, 29, 30, 32, 36, 44 , 45

Warm Up

1) Simplify $\frac{2}{a-2}+\frac{2}{a-3}$

Notes

Example 1: Solve $\frac{x+1}{3(x-2)}=\frac{5x}{6}+\frac{1}{x-2}$

Example 2: Solve $\frac{r+2}{2r+1}=\frac{r}{3}+\frac{3}{4r+2}$

Guided Practice p581 #8-10

Class work p582 #19, 22, 23, 25, 27, 30

Directions: Answer the following question(s).

1 Determine any vertical asymptotes for the function $g(x) = \dfrac{x^2 - 1}{x + 1}$.

A. $x = 1$

B. $x = -1$

C. There is not a vertical asymptote for the function $g(x)$

D. $x = 1$; $x = -1$

2 Determine any horizontal asymptote for the function $f(x) = \dfrac{6x - 1}{3x + 3}$.

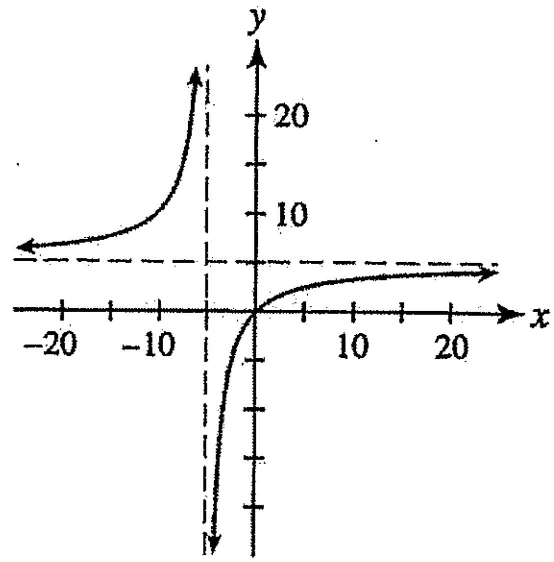
A. $y = -\dfrac{1}{3}$

B. There is not a horizontal asymptote for $f(x)$

C. $y = 6$

D. $y = 2$

3 Given the graph of r(x) below, determine any horizontal and vertical asymptote(s).



A. Horizontal Asymptote: y = 5
Vertical Asymptote: x = -5

B. Horizontal Asymptote: x = -5
Vertical Asymptote: y = 5

C. Horizontal Asymptote: y = 1
Vertical Asymptote: x = -1

D. Horizontal Asymptote: y = 5
Vertical Asymptote: x = 5

Directions: Answer the following question(s).

4 To estimate the volume of timber in a given area of forest, formulas have been developed to find the amount of wood contained in a tree with height h in feet and diamter d in inches. One study concluded that the volume, V, of wood in a tree varies jointly with the 1.12 power of h and the 1.98 power of d. Which of the following equations represents this situation?

A. $V = kh^{1.12}d^{1.98}$

B. $Vk = h^{1.12}d^{1.98}$

C. $V = \frac{h^{1.12}d^{1.98}}{k}$

D. $V = k^2h^{1.12}d^{1.98}$

5 The rate of vibration of a string under constant tension is inversely proportional to the length of the string. Which of the following equations represents the vibration rate of a string, V, as a function of its length, L?

A. $V = kL$

B. $V = \frac{k}{L}$

C. $V = \frac{L}{k}$

D. $L = \frac{V}{k}$

6 The time, T, required for a pendulum to swing back and forth once is called its period. The length, L, of a pendulum is directly proportional to the square of T. A 2-foot pendulum has a 1.63-second period. Find the constant of proportionality k. Round to the nearest hundreth.

A. 0.75

B. 1.57

C. 1.33

D. 0.64

7 If y varies inversely as x and y = 3 when x = 4, find y when x = 15.

A. $\frac{45}{4}$

B. $\frac{5}{4}$

C. 20

D. $\frac{4}{5}$

Directions: Answer the following question(s).

8 If y varies jointly as x and the cube of z, and y = 16 when x = 4 and z = 2, find y when x = -8 and z = -3.

A. -144

B. 2

C. 48

D. 108

9 Simplify the expression

$$\frac{1}{r+5} \cdot \frac{r^2-25}{r-10}$$

A. $\frac{r-5}{r-10}$

B. $\frac{r}{4(r-2)}$

C. $r+4$

D. 1

10 Simplify the expression.

$$\frac{r+7}{30r+100} \cdot \frac{12r+40}{r+7}$$

A. $\frac{4}{3}$

B. $\frac{6}{r-9}$

C. $\frac{9}{8}$

D. $\frac{2}{5}$

11 Simplify the expression.

$$\frac{5x^3-20x^2}{x^2-12x+32} \div \frac{1}{x-8}$$

A. $\frac{3}{5}$

B. $5x^2$

C. $\frac{5x^2}{x+1}$

D. $\frac{7x^2}{4}$

Directions: Answer the following question(s).

12 Simplify the expression

$$\frac{7}{a^2-13a+36} \div \frac{1}{a-9}$$

A. $a-1$

B. $\frac{a-7}{7a^2}$

C. $\frac{3}{a-7}$

D. $\frac{7}{a-4}$

13 Simplify the expression.

$$\frac{n+3}{n-5} + \frac{2n}{3}$$

A. $\frac{2n^2+9n-15}{3(n-5)}$

B. $\frac{2n^2-6n+5}{3(n-5)}$

C. $\frac{2n^2+11n-15}{3(n-5)}$

D. $\frac{2n^2-7n+9}{3(n-5)}$

14 Simplify the expression.

$$\frac{5}{n-1} + \frac{4}{n+2}$$

A. $\frac{9}{(n-1)(n+2)}$

B. $\frac{20}{(n-1)(n+2)}$

C. $\frac{9n+6}{(n-1)(n+2)}$

D. $\frac{9n-6}{(n-1)(n+2)}$

15 Simplify the expression.

$$\frac{3k}{k+2} - \frac{3}{k-3}$$

A. $\frac{3k-3}{(k+2)(k-3)}$

B. $\frac{k}{(k+2)(k-3)}$

C. $\frac{3k^2-12k-6}{(k-3)(k+2)}$

D. $\frac{4k^2-15k-6}{(k-3)(k+2)}$

Continue: Turn to the next page.

Directions: Answer the following question(s).

16 Simplify the expression.

$$\frac{5n}{5n-3} - \frac{6}{n+1}$$

A. $\frac{3}{2}$

B. $\frac{5n-6}{(5n-3)(n+1)}$

C. $\frac{5n^2-25n+18}{(5n-3)(n+1)}$

D. $\frac{-11n^2+33n+9}{(5n-3)(n+1)}$

17 Solve the equation. Remember to verify your solutions.

$$\frac{n+1}{3n^2} = \frac{n-3}{3n^2} + \frac{1}{n}$$

A. 2

B. 5

C. $\frac{4}{3}$

D. $\frac{5}{6}$

18 Solve the equation. Remember to verify your solutions.

$$\frac{1}{m} = \frac{m-1}{4m^2-5m} - \frac{m+6}{4m^2-5m}$$

A. $-\frac{3}{4}$

B. $-\frac{1}{2}$

C. 3

D. -3

19 Given the table for $f(x)$ below, find $f^{-1}(3)$

x	f(x)
1	3
2	5
3	7
7	-3

A. 1

B. 7

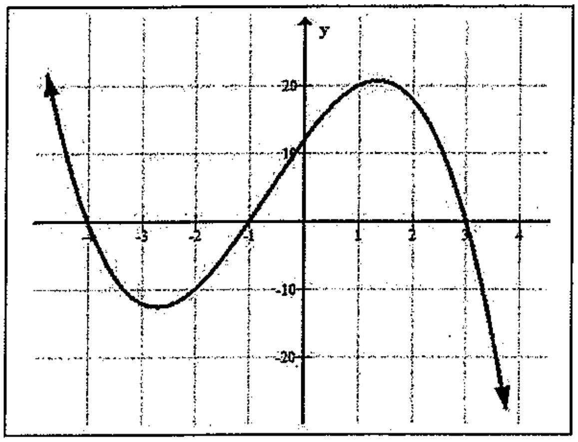
C. 5

D. -3

Continue: Turn to the next page.

Directions: Answer the following question(s).

20 Write the complete factored form of the polynomial in the graph below. The leading coefficient is either 1 or -1 and has integer zeros.



A. $y = (x - 4)(x - 1)(x + 3)$

B. $y = -(x - 4)(x - 1)(x + 3)$

C. $y = (x + 4)(x + 1)(x - 3)$

D. $y = -(x + 4)(x + 1)(x - 3)$

Stop: You have finished the assessment.

Warm-Up

Solve each equation or inequality. Check your solutions. *(Lesson 9-6)*

76. $\frac{15}{p} + p = 16$

77. $\frac{s-3}{s+4} = \frac{6}{s^2-16}$

Notes

Example 1: Graph $y = 2^x$. Then state the function's domain and range.

Key Concept ***Exponential Growth and Decay***

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

Example 2 *Identify Exponential Growth and Decay*

Determine whether each function represents exponential *growth* or *decay*.

Function	Exponential Growth or Decay?
a. $y = \left(\frac{1}{5}\right)^x$	
b. $y = 3(4)^x$	
c. $y = 7(1.2)^x$	

Example 3: In 1983, there were 102,000 farms in Minnesota, but by 1998, this number had dropped to 80,000.

(a) Write an exponential function of the form y = abx that can be used to model the farm population y of Minnesota. Write the function in terms of the number of years since 1983.

(b) Suppose the number of farms in Minnesota continues to decline at the same rate. Estimate the number of farms in 2010.

Example 4: Solve each equation

(a) $3^{2n+1} = 81$

(b) $4^{2x} = 8^{x-1}$

Class work

Sketch the graph of each function. Then state the function's domain and range.

21. $y = 2(3)^x$

22. $y = 5(2)^x$

23. $y = 0.5(4)^x$

24. $y = 4\left(\frac{1}{3}\right)^x$

25. $y = -\left(\frac{1}{5}\right)^x$

26. $y = -2.5(5)^x$

Determine whether each function represents exponential *growth* or *decay*.

27. $y = 10(3.5)^x$

28. $y = 2(4)^x$

29. $y = 0.4\left(\frac{1}{3}\right)^x$

30. $y = 3\left(\frac{5}{2}\right)^x$

31. $y = 30^{-x}$

32. $y = 0.2(5)^{-x}$

Write an exponential function whose graph passes through the given points.

33. $(0, -2)$ and $(-2, -32)$

34. $(0, 3)$ and $(1, 15)$

35. $(0, 7)$ and $(2, 63)$

36. $(0, -5)$ and $(-3, -135)$

37. $(0, 0.2)$ and $(4, 51.2)$

38. $(0, -0.3)$ and $(5, -9.6)$

Solve each equation or inequality. Check your solution.

45. $3^{n-2} = 27$

46. $2^{3x+5} = 128$

47. $5^{n-3} = \frac{1}{25}$

Warm-Up

Write an exponential function whose graph passes through the given points (0, 3) and (1, 15)

Notes

Example 1: In 1986, Kayla received $30.00 from her grandparents for her fifth birthday. Her mother deposited it into a bank account for her. Both Kayla and her other forgot about the money and made no further deposits or withdrawals. The table to below shows the account balance for several years.

Elapsed Time (years)	Balance
0	$30.00
5	$41.10
10	$56.31
15	$77.16
20	$105.71
25	$144.83
30	$198.43

(a) Use a graphing calculator to enter the data and draw a scatter plot that shows how the account balance is related to time.

(b) If Kayla discovers the account with the birthday money on her 50th birthday, how much will she have in the account?

Example 2: According to the World Almanac, the population per square mile in the United States has changed dramatically over a period of years.

Year	People Per Square Mile	Year	People Per Square Mile
1790	4.5	1900	21.5
1800	6.1	1910	26.0
1810	4.3	1920	29.9
1820	5.5	1930	34.7
1830	7.4	1940	37.2
1840	9.8	1950	42.6
1850	7.9	1960	50.6
1860	10.6	1970	57.5
1870	10.9	1980	64.0
1880	14.2	1990	70.3
1890	17.8		

(a) Use a graphing calculator to draw a scatter plot of the data. Then calculate and graph the curve of best fit that shows how the year is related to the number of people per square mile. Use an exponential regression for this example.

(b) Write the equation of best fit. Write a sentence that describes the fit of the graph of the data.

(c) Based on the model estimate the population density for 2000.

(d) Do you think there are any other types of equations that would be good models for this data? Why or why not?

(e) What event occurred between 1800 and 1810 that would account for the sudden decrease in population per square mile?

Warm-Up

Solve each equation by using the method of your choice. Find exact solutions.
(Lesson 6-5)

80. $9y^2 = 49$

81. $2p^2 = 5p + 6$

Notes

Key Concept — ***Logarithm with Base b***

- **Words** Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.
- **Symbols** Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.

Example 1: Write each equation in exponential form.

(a) $\log_8 1 = 0$

(b) $\log_2 \frac{1}{16} = -4$

Example 2: Write each equation in logarithmic form.

(a) $10^3 = 1000$

(b) $9^{\frac{1}{2}} = 3$

Example 3: Evaluate each logarithmic expression

(a) $\log_2 16$

(b) $\log_3 27$

(c) $\log_6 6^8$

(d) $3^{\log_3 (4x-1)}$

Example 4: Solve the following logarithmic equations/inequalities.

(a) $\log_4 n = \frac{5}{2}$

(b) $\log_5 x < 2$

(c) $\log_5 (p^2 - 2) = \log_5 p$

(d) $\log_{10}(3x-4) < \log_{10}(x+6)$

Class work

Write each equation in logarithmic form.

21. $8^3 = 512$

22. $3^3 = 27$

23. $5^{-3} = \frac{1}{125}$

24. $\left(\frac{1}{3}\right)^{-2} = 9$

25. $100^{\frac{1}{2}} = 10$

26. $2401^{\frac{1}{4}} = 7$

Write each equation in exponential form.

27. $\log_5 125 = 3$

28. $\log_{13} 169 = 2$

29. $\log_4 \frac{1}{4} = -1$

30. $\log_{100} \frac{1}{10} = -\frac{1}{2}$

31. $\log_8 4 = \frac{2}{3}$

32. $\log_{\frac{1}{5}} 25 = -2$

Evaluate each expression.

33. $\log_2 16$

34. $\log_{12} 144$

35. $\log_{16} 4$

36. $\log_9 243$

37. $\log_2 \frac{1}{32}$

38. $\log_3 \frac{1}{81}$

39. $\log_5 5^7$

40. $2^{\log_2 45}$

41. $\log_{11} 11^{(n-5)}$

42. $6^{\log_6 (3x+2)}$

43. $\log_{10} 0.001$

44. $\log_4 16^x$

Solve each equation or inequality. Check your solutions.

47. $\log_9 x = 2$

48. $\log_2 c > 8$

49. $\log_{64} y \leq \frac{1}{2}$

50. $\log_{25} n = \frac{3}{2}$

51. $\log_{\frac{1}{7}} x = -1$

52. $\log_{\frac{1}{3}} p < 0$

53. $\log_2 (3x - 8) \geq 6$

54. $\log_{10} (x^2 + 1) = 1$

55. $\log_b 64 = 3$

56. $\log_b 121 = 2$

57. $\log_5 5^{6n+1} = 13$

58. $\log_5 x = \frac{1}{2}$

59. $\log_6 (2x - 3) = \log_6 (x + 2)$

60. $\log_2 (4y - 10) \geq \log_2 (y - 1)$

61. $\log_{10} (a^2 - 6) > \log_{10} a$

62. $\log_7 (x^2 + 36) = \log_7 100$

$\log_3 27 = x$

Lesson 7.4 Warm Up

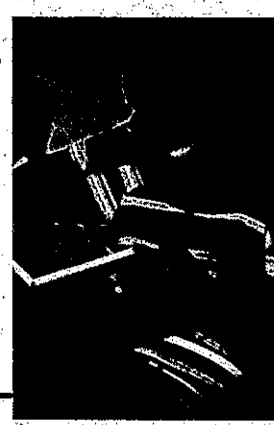
$\log_2(4x-2)=3$

$\log_3(2x+7)=\log_3(5x-14)$

1

Essential Question

- What are the properties of logarithms and how do we apply them?
- Vocabulary
  - Logarithm
- Quiz!



2

Can you evaluate these?

$$\log_4 64 + \log_2 2$$

$$\log_5 25 + \log_3 9$$

$$\log_6 9 + \log_6 4$$

3

Properties of Logarithms

- Let b, u, and v be positive numbers such that b≠1.
- **Product property:**

$$\log_b uv = \log_b u + \log_b v$$

- **Quotient property:**

$$\log_b u/v = \log_b u - \log_b v$$

- **Power property:**

$$\log_b (u)^n = n \log_b u$$

4

Expanding Logarithms

- You can use the properties to expand logarithms.
- $\log_2 \frac{7x^3}{y} =$

5

More Practice

- Expand:
- log 5mn =
- Expand:
- $\log_5 8x^3 =$

6

Condensing Logarithms

- log 6 + 2 log2 – log 3 =

7

More Practice

- Condense:
- $\log_5 7 + 3 \cdot \log_5 t =$
- Condense:
- $3\log_2 x - (\log_2 4 + \log_2 y) =$

8

Practice

$\log x^2 y$

$\log(xy)^3$

$\log \frac{ac}{b}$

$\log r^{\frac{1}{3}} t$

$\log x^5 y^2$

$\log \frac{x^3}{y}$

$\log(xy)^2$

$\log \sqrt[4]{xyz}$

$\log \frac{xy}{z}$

$\log \frac{x^2}{y^5}$

9

Lesson
7.5
Warm Up

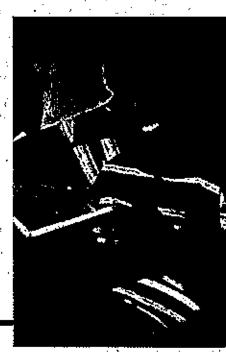
1) $\log_2(4x-3)=4$

2) $\log_6(5x+4)=\log_6(10x-3)$

1

Essential Question

- How do we solve logarithmic & exponential equations and apply logarithms to real-world problems?
- Vocabulary
  - Logarithm
- Quiz
- Test!



2

IMPORTANT

You can **ONLY cross** out the LOGS **when** there is a **SINGLE** log on the **left** side and a **SINGLE** log on the right side.

$$\log_6(5x+4)=\log_6(10x-3)$$

You **CAN NOT** cross all of these **logs** out below. You have to **CONDENSE** it so that there is a SINGLE log on the left side and a SINGLE log on the right side

$$\log_5 x-\frac{1}{2}\log_5 16=\log_5 25$$

3

Solving Log Equations

$$\log_5 x+\log_5 16=\log_5 25$$

4

Examples

$$\log(x+4)-\log 5=\log 2$$

5

Examples

$$\log_6(x-4)+\log_6 8=2$$

6

Examples

$$\log_4(x-1)+\log_4 3=3$$

7

Aviation

- The altitude of an aircraft is in part affected by the outside air pressure and can be determined by the equation below, where "h" is the altitude in miles, "P" is the air pressure outside the aircraft, and "B" is the air pressure at sea level. Normally B=14.7 pounds per square inch.

(a) Suppose the air pressure outside an airplane is 10.3. What is the altitude of the plane?

$$h=-\frac{100}{9}\log\frac{P}{B}$$

8

Interest

- Mr. Kelly invests $2,000 at a rate of 4.5% interest compounded yearly. Approximately how many years will he have to invest his money in order to have $10,000?

$$A = P(1+r)^t$$

9

Solve for x

$$2^x = 24$$

10

Example #1

$$10^{5x} - 12 = 191$$

11

Example #2

$$2 \cdot 4^{x-3} = 9$$

12

Example #3

$$5e^{2x+1} = 127$$

13

Lesson 7.6
Warm Up - Solve

$\log_2 x = 2\log_2 4 - 4\log_2 3$

$\log_2 x + \log_2 5 = 3$

1

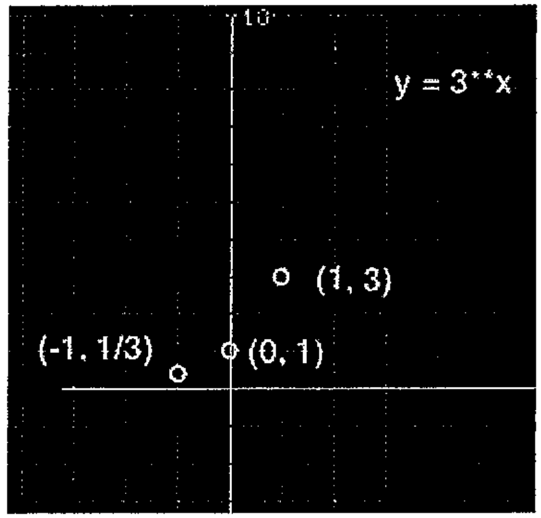
Essential Question

- How do we graph logarithmic functions?
- Vocabulary
  - Logarithm
  - Inverse
  - Domain & Range
- Test!

2

General Form of Exponential Function $y = b^x$ where $b > 1$

- Domain: All reals
- Range: $y > 0$
- Growth
- y-intercept: (0, 1)



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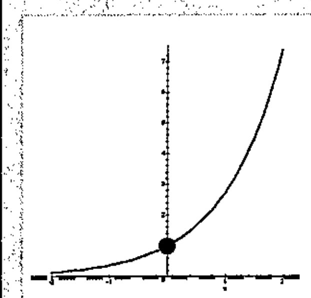
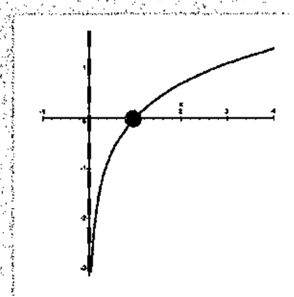
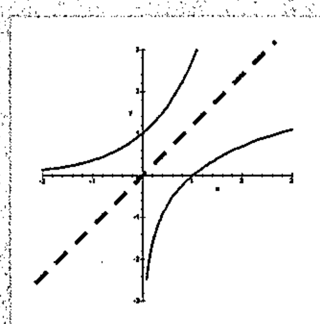


Exponential Graph



Logarithmic Graph

Graphs of inverse functions are reflected about the line $y = x$



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Relationships of Exponential ($y = b^x$) *& Logarithmic* ($y = \log_b x$) *Functions*

- $y = b^x$
- Domain: All Reals
- Range: $y > 0$
- Asymptote $y=0$
- y-intercept: (0, 1)

- $y = \log_b x$ is the **inverse of $y = b^x$**
- Domain: $x > 0$
- Range: All Reals
- x-intercept: (1, 0)
- Asymptote $x=0$

5

Asymptote (think opposite)

$$f(x) = \log_2(x-4)$$

$$f(x) = \log_2(x+3)$$

$$f(x) = \log_2(x-6)$$

$$f(x) = \log_2(x+9)$$

6

You can graph a logarithmic function by constructing a table of values.

$$f(x) = \log_3(x+1)$$

7

Guided Practice

$$f(x) = \log_2(x-4)+1$$

8

Guided Practice

$f(x) = \log_3(x-1) - 2$

9

Practice

$\log_5 249 =$

$4^{x+2} = 90$

$\log(3x+1) = \log(4x-7)$

$\log_2(4x+1) = 5$

10

Algebra 2 Name______________________ ID: 1

Test 7 - Form A

Date________________ Period____

Divide.

1) $(7x^2 + x^3 + 17x + 11) \div (3 + x)$

Factor each completely.

2) $5n^3 - 8n^2 - 20n + 32$

Find all zeros.

3) $f(x) = x^3 + 2x + 12$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

4) $5, \ i, \ -i$

Perform the indicated operation.

5) $g(x) = x^2 - 2x$
$f(x) = -2x - 4$
Find $g(f(x))$

Find the inverse of the function.

6) $f(x) = 5x + 10$

Expand each logarithm.

7) $\log_6 \sqrt[3]{x \cdot y \cdot z}$

8) $\log_2 (a^4 b^3)$

Condense each expression to a single logarithm.

9) $\log_4 w + \frac{\log_4 x}{2} + \frac{\log_4 y}{2} + \frac{\log_4 z}{2}$

10) $\log_9 w + 4\log_9 u - 4\log_9 v$

Solve each equation.

11) $\log_5 (r + 9) = \log_5 (3r - 9)$

12) $\log_2 (10x + 8) = 2$

13) $\log_3 (x - 9) + \log_3 4 = \log_3 71$

14) $\log_8 (x+8) - \log_8 2 = 2$

Solve each equation. Round your answers to the nearest ten-thousandth.

15) $3 \cdot 10^{5-4p} = 9$

16) $e^{-10x-1} - 6 = 61.5$

17) The formula for finding the amount of an investment with interest compounded yearly for t years is: $A = P(1 + r)^t$, where A is the amount accrued, P is the principal, and r is the rate. James invests $2,345 at a rate of 5.13% interest compounded yearly. Approximately how many years will he have to invest his money in order to have $5,000? Round your answer to the nearest year.

18) The altitude of an aircraft is in part affected by the outside air pressure and can be determined by the equation $h = -\frac{100}{9}\log\frac{P}{B}$, where "h" is the altitude in miles, "P" is the air pressure outside the aircraft, and "B" is the air pressure at sea level. Normally B = 14.7 pounds per square inch. Suppose the air pressure outside an airplane is 10.3. What is the altitude of the plane?

Simplify.

19) $\frac{7}{3 + 2i}$

Solve each equation.

20) $-11 = 5 - 2(-23 - n)^{\frac{3}{2}}$