AP Calc AB

AP Calculus AB

Packet #2

Topics Covered:

Unit 8 – AP Exam Review

This packet consists of daily Notes with examples and practice problems. Solutions to these worksheets are found on "Schoology." There is 1 Test in this packet. You are to take this test and submit for grading. Your score on this test will determine the number of awarded points (out of 5 max points).

When a certain grocery store opens, it has 60 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \; for \; 0 < t \leq 12,$$

Where f(t) is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln(t^2 + 2t) \; for \; 3 < t \leq 12,$$

Where g(t) is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 3 hours the store is open?

(b) Find f′(6). Using correct units, explain the meaning of f′(6) in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time t = 4? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time t = 7?

AP Calculus　　　　Test 8 FRQ #2　　　　No Calculator

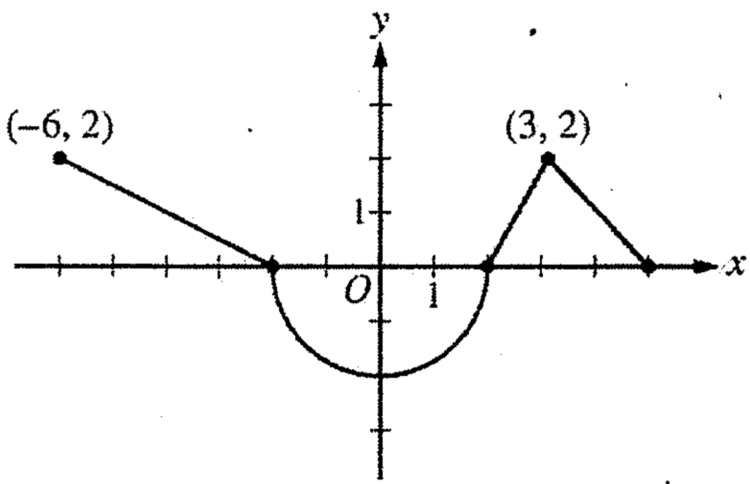
Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln(t^2 - 2t + 10)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position x = 7 at time t = 0.

(a) For $0 \le t \le 8$, when is particle P moving to the right?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time t = 6. Is the speed of particle Q increasing, decreasing, or neither at time t = 6? Explain your reasoning.

(d) Find the position of particle Q the second time it changes direction.



Graph of f'

The function f is differentiable on the closed interval [-6, 5] and satisfies f(-2) = 9. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of f(-6) and f(5).

(b) On what intervals is f decreasing? Justify your answer.

(c) Find the absolute maximum value of f on the closed interval [-6, 5]. Justify your answer.

(d) For each of f''(4) and f''(3), find the value or explain why it does not exist.

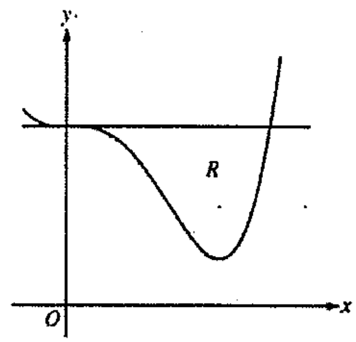
Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 20$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where A(t) is measured in pounds and t is measured in days.

(a) Find the average rate of change of A(t) over the interval $0 \le t \le 20$. Indicate units of measure.

(b) Find the value of A '(20). Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 20$

(d) For t > 20, L(t), the linear approximation to A at t = 20, is a better model for the amount of grass clippings remaining in the bin. Use L(t) to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.



Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line y = 4, as shown in the figure above.

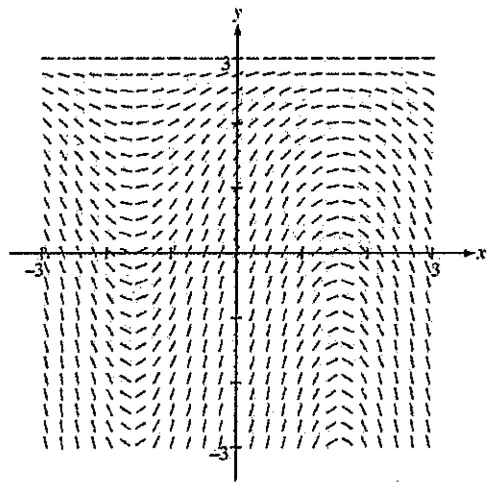
(a) Find the volume of the solid generated when R is rotated about the horizontal line y = 4.

(b) Find the volume of the solid generated when R is rotated about the horizontal line y = -3.

(c) The vertical line x = k divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value of k.

Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let y = f(x) be the particular solution to the differential equation with initial condition f(0) = 1. The function is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point (0, 1)



(b) Write an equation for the line tangent to the solution curve at the point (0, 1). Use the equation to approximate f(0.2)

(c) Find y = f(x), the particular solution to the differential equation with the initial condition f(0) = 1.

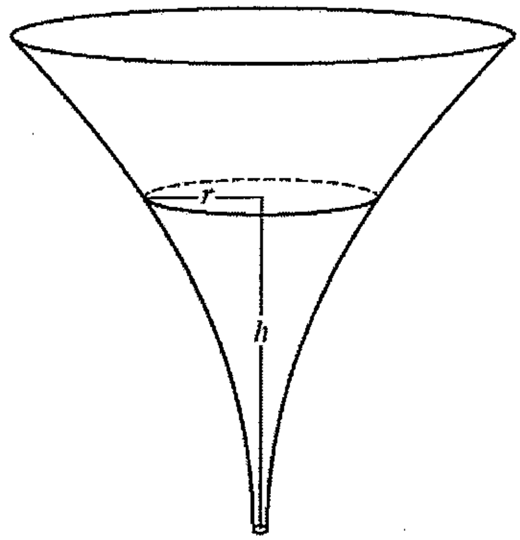
x	f(x)	f'(x)	g(x)	g'(x)
1	-6	3	2	8
2	2	-2	-3	1
3	8	7	6	2
6	4	5	3	-1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let k(x) = f(g(x)). Write an equation for the line tangent to the graph of k at x = 6.

(b) Let $h(x)=\frac{g(x)}{f(x)}$. Find h'(2).

(c) Evaluate $\int_1^2 f''(3x)dx$.



The inside of a funnel of height 5 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3+h^2)$, where $0 \le h \le 5$. The units of r and h are inches.

(a) Find the average value of the radius of the funnel.

(b) Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is h = 3 inches, the radius of the surface of the liquid is decreasing at a rate of 1/10 inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

For $t \geq 0$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = 1 + 2\sin\left(\frac{t^2}{2}\right)$.

The particle is at position x = 3 at t = 5.

(a) At time t = 5, is the particle speeding up or slowing down?

(b) Find all times t in the interval 0 < t < 4 when the particle changes direction. Justify your answer.

(c) Find the position of the particle at time t = 0.

(d) Find the total distance the particle travels from time t = 0 to t = 4.

t (hours)	0	1	3	6	8
R(t) (liters/hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by R(t) liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of R(t) are shown in the table above. At time t = 0, there are 40,000 liters of water in the tank.

(a) Estimate R'(7). Show the work that leads to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

AP CALCULUS AB

TEST 8

1) What is the total area of the regions between the curves $y = 6x^2 - 18x$ and $y = -6x$ from x = 1 to x = 3?

2) What is the value of $y'' - 3y' - 15$ given the function $y = e^{3x} - 5x + 7$?

3) Let y = f(x) be a twice-differentiable function such that f(1) = 2 and $\frac{dy}{dx} = y^3 + 3$. What is the value of $\frac{d^2y}{dx^2}$ at x = 1?

4) Let y = f(x) be the particular solution to the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$ with initial condition f(0) = -2. What is the expression for f(x)?

5) If $y = (x^3 + 1)^2$, then dy/dx =

6) $\int_0^1 e^{-4x} dx$

7) If $y=\dfrac{2x+3}{3x+2}$, then dy/dx =

8) $\int x^2\cos(x^3)dx$

9) $\int (3x+1)^5\,dx$

10) What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point (3, 2)?

11) A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

12) $\int_1^2 \frac{x-4}{x^2} dx$

13) The Volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of ½ centimeters per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

14) At what value(s) of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

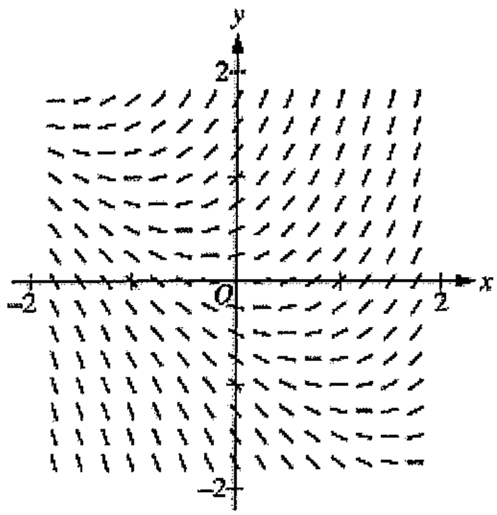
15) What is the absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval [-2, 4]?

16) $\int \frac{x^2}{e^{x^3}} dx$

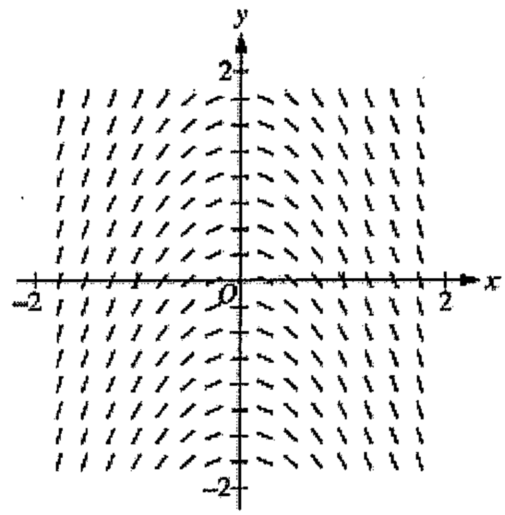
20)

Which of the following could be a slope field for the differential equation $\frac{dy}{dx} = x^2 + y$?

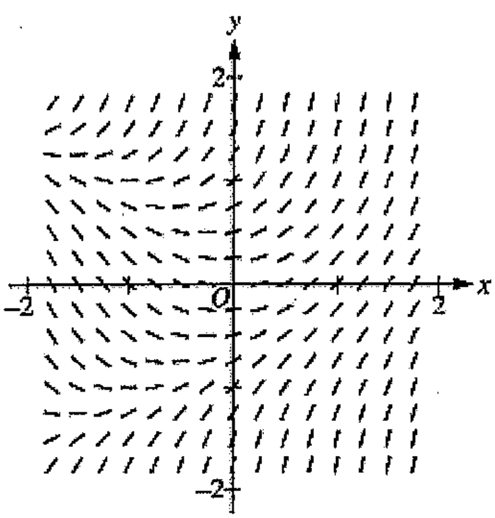
(A)



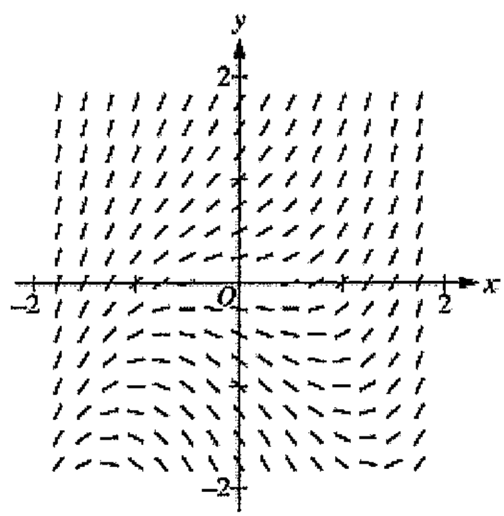
(B)



(C)



(D)



21) $\int 7xe^{3x^2}\,dx$

22) A solid is generated when the region in the first quadrant bounded by the graph of $y = 1 + \sin^2 x$, the line $x = \frac{\pi}{2}$, the x-axis, and the y-axis is revolved about the x-axis. Set up, but do not integrate, an integral that can be used to find its volume.

23) If $\frac{dy}{dx} = \frac{x^3 + 1}{y}$ and y = 2 when x = 1, then, when x = 2, y = ?

24) If $f(x) = x^2\sqrt{3x+1}$, the f'(x) =

25) Find the area of the region bounded by the parabolas $y = x^2$ and $y = 6x - x^2$