

Test 7 – Day 1 – Intro to Differential Equations

Warm Up

- 1) Where does the graph of $y = 5x^4 - x^5$ have an inflection point?
- 2) Find two nonnegative numbers x and y whose sum is 100 and for which xy^2 is a maximum.

Notes

A differential equation is any equation involving a derivative. We have solved some simple differential equations before when we were given the velocity at time t of a particle moving along the x -axis, and found the position function. A solution of a d.e. is any function that satisfies it. We call the antiderivative of a d.e. the “general solution” since it has an infinite number of solutions—one for each real value of C . If given an initial condition, we call this the “particular solution” since it gives a precise solution for the given situation.

A d.e. contains derivatives. A derivative gives information about the rate of change of a function. For example: If P is the size of a population at time t , then we can interpret the d.e. $\frac{dP}{dt} = 0.0325P$ as saying that at any time t the rate at which the population is growing is directly proportional (3.25%) to its population size at that time.

For the following problems, $a(t)$ denotes the acceleration function, $v(t)$ the velocity function, and $s(t)$ the position or height function at time t .

- 1) If $a(t) = 4t - 1$ and $v(1) = 3$ then $v(t)$ equals what?
- 2) If $a(t) = 20t^3 - 6t$, $s(-1) = 2$, and $s(1) = 4$, then $v(t)$ equals what?

Quiz Review

- 1) If $f(x) = 5x \ln x$ then $f'(x) =$
- 2) If $f(x) = (x-2)(x^2+1)^3$, then $f'(x) =$
- 3) If $h(x) = f(x)g(5x)$ then $h'(3) =$
- 4) If $f(x) = \sin^2 x$ then $f''(\pi) =$
- 5) $\lim_{x \rightarrow 0} \frac{\cos(\pi + x) + 1}{x} =$

Test 7 – Day 2 – Slope Fields

Warm Up

- 1) An isosceles right triangle with legs S has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?
- 2) The top of a 15-foot ladder rests against a vertical wall with the bottom of the ladder on level ground. The ladder is sliding down the wall at a constant rate of 2 feet per second. At what rate, in radians per second, is the acute angle between the bottom of the ladder and the ground changing at the instant the bottom of the ladder is 9 feet from the base of the wall?

Notes

In this section, we investigate d.e.'s by obtaining a slope field that approximates the general solution. We call the graph of a solution of a d.e. a solution curve. Graph each of the following d.e.'s.

1) $\frac{dy}{dx} = y$

2) $\frac{dy}{dx} = \frac{1}{x}$

3) $\frac{dy}{dx} = \frac{-x}{y}$

4) $y' = x + y$

Quiz Review

- 1) If $x + 2xy - y^2 = 2$ then at the point $(1, 1)$ $\frac{dy}{dx} =$
- 2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$
- 3) For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative minimum?
- 4) The function f is given by $f(x) = x^4 + x^2 - 2$. On what interval(s) is f decreasing?
- 5) If $f''(x) = x(x+2)(x-3)^2$, then the graph of f has inflection points when $x =$

Test 7 – Day 3 – Slope Fields

Warm Up

1) If $f(x) = \ln(\ln(2-x))$, then $f'(x) =$

2) Let f be a differentiable function with selected values in the table below. What is the average rate of change of f over the closed interval $0 \leq x \leq 10$?

x	0	2	4	6	8	10
f(x)	4	7	8	0	-16	-21

3) The radius of a sphere is increasing at a constant rate of 2 cm/sec. At the instant when the volume of the sphere is increasing at a rate of $32\pi \frac{\text{cm}^3}{\text{sec}}$, the surface area of the sphere is what?

4) If $f(x) = \sin(3x)$ then $f'(\frac{\pi}{9}) =$

Notes

1) What can be said about the general solution for each of the following d.e.'s?

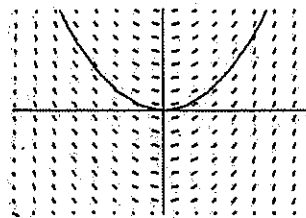
(A) $y' = \cos x$

(B) $\frac{dy}{dx} = 2x$

(C) $\frac{dy}{dx} = 3x^2 - 3$

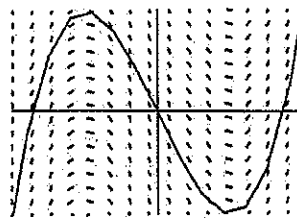
(D) $y' = -\frac{\pi}{2}$

Then, match each slope field with the proper d.e. from the following set.



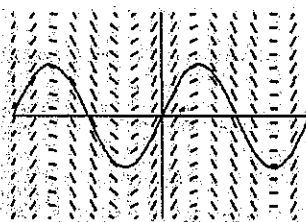
$[-2,2] \times [-2,2]$

Figure N9-3a



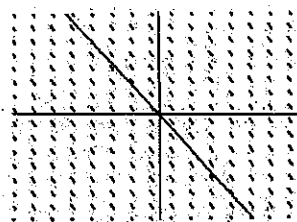
$[-2,2] \times [-2,2]$

Figure N9-3b



$[-2\pi, 2\pi] \times [-2,2]$

Figure N9-3c



$[-2,2] \times [-2,2]$

Figure N9-3d

Test 7 – Day 4 – Separating Variables

Warm Up

1) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{3 \cos x}{x - \frac{\pi}{2}} =$

2) Let f be a continuous and twice differentiable function and $f''(x) > 0$ for all x in the interval $[4, 5]$. Some of the values of f are shown in the table below. What is the approximate value of $f'(4.6)$?

x	4.5	4.6	4.7	4.8
$f(x)$	10.2	10.8	11.9	12.3

3) Given the equation $y = 3e^{-2x}$, what is an equation of the line normal to graph at $x = \ln 2$?

4) The velocity function of a moving particle on the x -axis is given as $v(t) = t^2 - 3t - 10$. For what positive values of t is the particle's speed increasing?

Notes

A first-order d.e. in x and y is separable if it can be written so that all the terms involving y are on one side and all the terms involving x are on the other.

1) Solve the d.e. $\frac{dy}{dx} = -\frac{x}{y}$ given the initial condition $y(0) = 2$.

Test Review

1) If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, solve the d.e.

2) The curve that passes through the point $(1, 1)$ and whose slope at any point (x, y) is equal to $\frac{3y}{x}$ has the equation?

3) If $\frac{dy}{dx} = e^y$ and $y = 0$ when $x = 1$, solve the d.e.

Test 7 – Day 5 – Separating Variables

Warm Up

1) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 5}}{3x - 1} =$

2) If $f(x) = \frac{3x}{(3x-1)^2}$, then $f'(x) =$

3) Given the equation $V = \frac{1}{3}\pi r^2(5-r)$, what is the instantaneous rate of change of V with respect to r at $r = 4$?

Notes

A first-order d.e. in x and y is separable if it can be written so that all the terms involving y are on one side and all the terms involving x are on the other.

1) If $\frac{ds}{dt} = \sqrt{st}$ and $t = 0$ when $s = 1$, find s when $t = 9$.

2. The population of a country is growing at a rate proportion to its population. If the growth rate per year is 4% of the current population, how long will it take for the population to double?

3) According to Newton's law of cooling, a hot object cools at a rate proportional to the difference between its own temperature and that of its environment. If a roast at room temperature 68°F is put into a 20°F freezer, and if, after 2 hours, the temperature of the roast is 40°F , what is the temperature after 5 hours? How long will it take for the temperature of the roast to fall to 21°F ?

AP Calculus Practice Test 7

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- 9) The radius of a sphere is increasing at a constant rate of 2 cm/sec. At the instant when the volume of the sphere is increasing at a rate of $32\pi \frac{\text{cm}^3}{\text{sec}}$, the surface area of the sphere is what?

- 10) If $f(x) = \sin(3x)$ then $f'(\frac{\pi}{9}) =$

11) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{3 \cos x}{x - \frac{\pi}{2}} =$

- 12) Let f be a continuous and twice differentiable function and $f''(x) > 0$ for all x in the interval $[4, 5]$. Some of the values of f are shown in the table below. What is the approximate value of $f'(4.6)$?

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15) If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, solve the d.e.

16) The curve that passes through the point $(1, 1)$ and whose slope at any point (x, y) is equal to $\frac{3y}{x}$ has the equation?

17) If $\frac{dy}{dx} = e^y$ and $y = 0$ when $x = 1$, solve the d.e.

18) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 5}}{3x - 1} =$

19) If $f(x) = \frac{3x}{(3x-1)^2}$, then $f'(x) =$

20) Given the equation $V = \frac{1}{3}\pi r^2(5-r)$, what is the instantaneous rate of change of V with respect to r at $r = 4$?

