

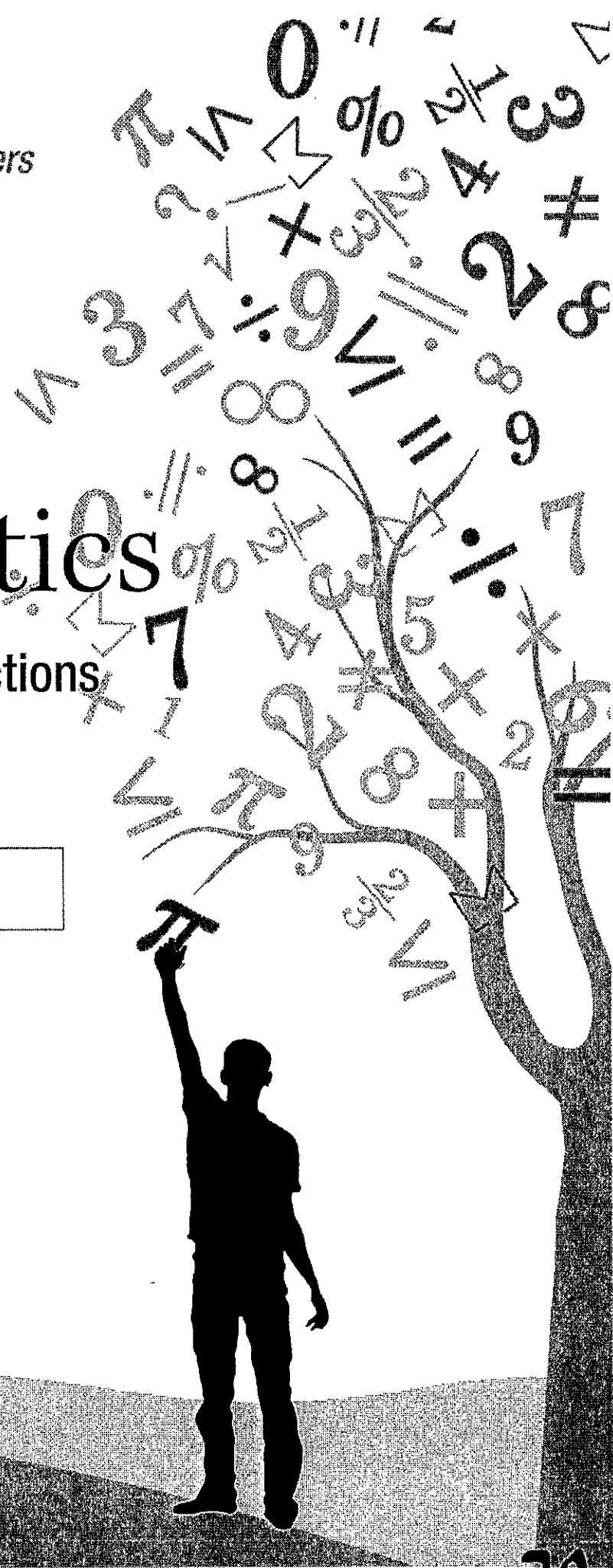
SREB Readiness Courses
Transitioning to college and careers

College Readiness Mathematics

Unit 7 . Exponential Functions
Student Manual

Name

Harper
L. Brown



Unit 7 . Exponential Functions

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Task #1: Growth vs. Decay

For each of the situations below, set up a table, write a general formula, and sketch a graph to represent the output in terms of the input.

1. North Dakota has recently had the fastest growing population out of all 50 states. On Jan 1, 2013, the population of North Dakota was 700,000 people. North Dakota's population has been growing by 5% per year. Express North Dakota's population, N , in terms of years since 2013, t (use data from your state, if applicable).
- exponential growth*

2. An air freshener starts with 30 grams of fluid, and the amount of fluid decreases by 12% per day. Express the amount of grams of freshener, Q , that remains t days after it has begun being used.

Task #2: Linear or Exponential?

1. In (a)–(e), say whether the quantity is changing in a linear or exponential fashion.

a. A savings account, which earns no interest, receives a deposit of \$723 per month.

b. The value of a machine depreciates by 17% per year.

c. Every week, $\frac{9}{10}$ of a radioactive substance remains from the beginning of the week.

d. A liter of water evaporates from a swimming pool every day.

e. Every 124 minutes, $\frac{1}{2}$ of a drug dosage remains in the body.

(Source: Illustrative Mathematics)

2. The functions below represent exponential growth or decay. What is the initial quantity? What is the growth rate? Is this growth or decay and how do you know? Make a rough sketch of the graph of the function and write a story problem to go with each equation.

a. $P=8(1.23)^t$

b. $Q=3.1(0.78)^t$

c. $y=3^{t/2}$

d. $w = \left(\frac{3}{2}\right)^t$

e. $P=10(3)^{t/2}$

Task #3: Population and Food Supply

The population of a country is initially two million people and is increasing at a rate of 4% per year. The country's annual food supply is initially adequate for four million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

- a. Based on these assumptions, in approximately what year will this country first experience shortages of food?

Pop. $y = ab^x$ $y = 2(1 + 0.04)^x$
 $y = 2(1.04)^x$

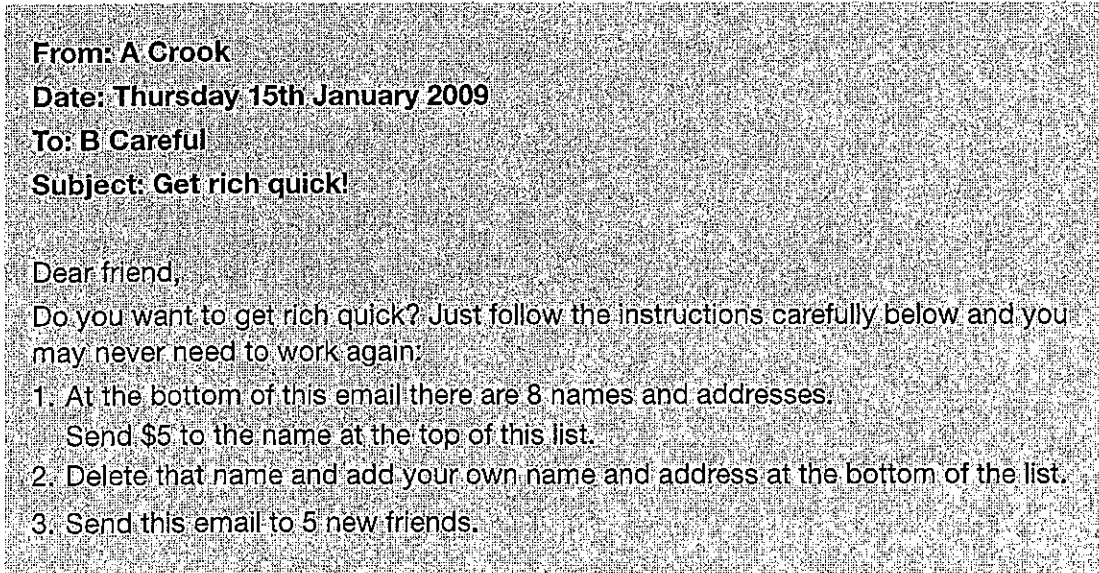
- b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

- c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

(Source: Illustrative Mathematics)

Task #4: Ponzi Pyramid Schemes

Max has received this email. It describes a scheme for making money.



If that process goes as planned, how much money would be sent to Max? Show your calculations.

(<http://map.mathshell.org/materials/download.php?fileid=808>)

“Ponzi” Pyramid Schemes: (continued)

2. What could possibly go wrong? Explain your answer clearly.

3. Why do they make Ponzi schemes like this illegal?

Task #5: Snail Invasion

In 1966, a Miami boy smuggled three Giant African Land Snails into the country. His grandmother eventually released them into the garden, and in seven years there were approximately 18,000 of them. The snails are very destructive and had to be eradicated. According to the USDA, it took 10 years and cost \$1 million to eradicate them.

- a. Assuming the snail population, $P(t)$, grows exponentially, write an expression for it in terms of the number, t , of years since their release.

- b. By what percent did snail population grow each year?

- c. By what percent did the snail population grow each month?

- d. Using a calculator or technology, determine how long does it take for the population to double?

- e. *(Optional for additional challenge)* Assuming the cost of eradicating the snails is proportional to the population, how much would it have cost to eradicate them if
 - i. *They had started the eradication program a year earlier?*

 - ii. *They had let the population grow unchecked for another year?*

(Source: Illustrative Mathematics)

Task #6: Facebook Users

The number of Facebook users worldwide reached one billion on October 4, 2012. Behind India and China, Facebook would be the third largest country in the world (larger than the US!) On April 24, 2012 there were 800 million Facebook users worldwide. Find a formula for the total number of Facebook users, N (in billions of users), t days after Jan 1, 2012. This means January 1 is $t = 0$, January 2 is $t = 1$, ..., and December 31 is $t = 365$. (Note 2012 was a leap year which is why December 31 is $t = 365$. In a non-leap year December 31 is $t = 364$).

Task #7: Forms of Exponential Expressions

Four physicists describe the amount of a radioactive substance, Q in grams, left after t years:

- $Q = 300e^{-0.0577t}$
- $Q = 300\left(\frac{1}{2}\right)^{t/12}$
- $Q = 300 \times 0.9439^t$
- $Q = 252.290 \times (0.9439)^{t-3}$

a. Show that the expressions describing the radioactive substance are all equivalent (using appropriate rounding).

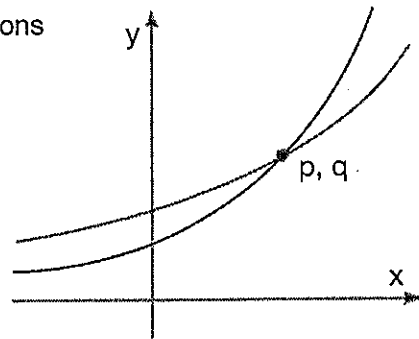
b. What aspect of the decay of the substance does each of the formulas highlight

(<http://www.illustrativemathematics.org/illustrations/1305>)

(Source: Illustrative Mathematics)

Task #8: Exponential Functions

The figure to the right shows the graphs of the exponential functions $f(x) = c \cdot 3^x$ and $g(x) = d \cdot 2^x$, for some numbers $c > 0$ and $d > 0$. They intersect at the point (p, q) .



a. Which is greater, c or d ? Explain how you know.

b. Imagine you place the tip of your pencil at (p, q) and trace the graph of g out to the point with x -coordinate $p + 2$. Imagine I do the same on the graph of f . What will be the ratio of the y -coordinate of my ending point to the y -coordinate of yours?

(Source: Illustrative Mathematics)

Task #9: Illegal Fish

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled $P(x) = 5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

- Exactly how many fish did the fisherman release into the lake?

- Find b if you know the lake contains 33 fish after eight weeks. Show step-by-step work.

- Instead, now suppose that $P(x) = 5b^x$ and $b = 2$. What is the weekly percent growth rate in this case? What does this mean in every-day language?

(Source: Illustrative Mathematics)

Task #10: Buying a Car

You wish to purchase a certain car. Two dealerships in town are selling the car for \$20,000. Both dealerships are unique in unusual finance offers. Rather than monthly payments, you are charged interest over time, yet you are expected to pay the car off (plus interest) in one lump sum payment at a date of your choosing. The dealerships don't want to deal with paper work and are really only interested in "loaning" you money with interest. However, each offers a different payment plan. You have discretion of when you want to pay off this car.

- A. No down payment needed or payments in the first year. When you do pay for the car, you will make one full payment for the car plus any interest accrued. This plan comes with a 12% interest/per year charge.
- B. No down payment needed. No fees or penalties for not making payments. Again, you will make one full payment for the car plus any interest accrued. This plan charges 1% interest per month.

As a group, decide what plan is better for your unique needs. Your presentation to your peers should include details about how much you will have to pay off at different times and how your group arrived at the decisions you made.

Task #11: Part 1: Saving for College

When you invest money in a bank account (and add interest to your balance), the same terminology and notation applies. For example, imagine parents of a newborn baby want to invest money today in order to pay for the child's college 18 years from now. They have \$10,000 of savings they wish to deposit all at once into one savings account, which they will withdraw from 18 years from now.

- A. Bank A advertises an APR of 6% with monthly compounding. (Think about how much of this interest is applied monthly.)
- B. Bank B advertises an EAR of 6%. This means 6% interest is accrued once each year.

Which bank has the better savings account? Create a model that shows what plan the parents should choose in order to save the maximum amount possible for college.

Part 2: Saving for College with the End in Mind

Imagine the parents wish to have \$150,000 in account A in 18 years, how much would they need to deposit today?

Task #12: Buying on Credit

If you charge \$500 on a credit card today, how much will the balance be in two years (assuming no additional fees) if the credit card has a 10% APR that is compounded—

- a. once a year?

- b. once a month?

- c. once a week?

If you need \$25,000 eight years from now, what is the minimum amount of money you need to deposit into a bank account that pays an annual percentage rate (APR) of 5% that is compounded—

- a. once a year?

- b. once a month?

- c. once a day?

Task #13: The Bank Account

Most savings accounts advertise an annual interest rate, but they actually compound that interest at regular intervals during the year. That means that, if you own an account, you'll be paid a portion of the interest before the year is up, and, if you keep that payment in the account, you will start earning interest on the interest you have already earned.

For example, suppose you put \$500 in a savings account that advertises 5% annual interest. If that interest is paid once per year, then your balance B after t years could be computed using the equation $B = 500(1.05)^t$, since you'll end each year with 100% + 5% of the amount you began the year with.

On the other hand, if that same interest rate is compounded monthly, then you would compute your balance after t years using the equation:

$$B = 500\left(1 + \frac{0.05}{12}\right)^{12t}$$

- a. Why does it make sense that the equation includes the term $\frac{0.05}{12}$?
That is, why are we dividing 0.05 by 12?

- b. How does this equation reflect the fact that you opened the account with \$500?

- c. What do the numbers 1 and $\frac{0.05}{12}$ represent in the expression $\left(1 + \frac{0.05}{12}\right)$?

- d. What does the "12t" in the equation represent?

(Source: Illustrative Mathematics)

Task #15: Monthly Deposits

If the family deposits \$500 each month into this account (6% APR compounded monthly), how much money will they have in the account—

a. one month later?

b. two months later?

c. six months later?

d. 18 years later?

Task #16: Retirement Planning

1. If you want to save \$750,000 for your retirement, you invest your money in a savings account that has an APR of 5% which is compounded each month. You are 20 years old and planning to retire at age 65, how much money do you need to deposit into the savings account each month in order to reach your retirement goal at age 65?

2. You are hired as a _____ (pick your career) and are offered an annual salary of _____ (research the average salary of your chosen career). You plan to contribute 2% of your paycheck each month to into a retirement account that with an APR of 7% compounded each month.

a. What is your monthly paycheck before taxes (contributions to retirement funds are typically taken out of the paycheck before taxes)?

b. How much money will have been saved if you work for 40 years at this job?

c. How much would you need to contribute each month in order to have \$500,000 when you retire?

d. Now assume that more realistically you are offered a raise of 2.5% each year. If you contribute 2% of your paycheck each month, how much will you have saved at the time of your retirement in 40 years?

Task #17: Car Purchase Options

You have three options to buy your new car (assume all of which have a 6% annual percentage rate that is being compounded monthly):

- A. Pay the entire cost of the car on the day of purchase.
- B. Payoff the car in 71 equal payments over 72 months (no money down).
- C. Make a down payment of 20% at the time of purchase. Then payoff the remaining value in 71 equal payments over **72 months.**